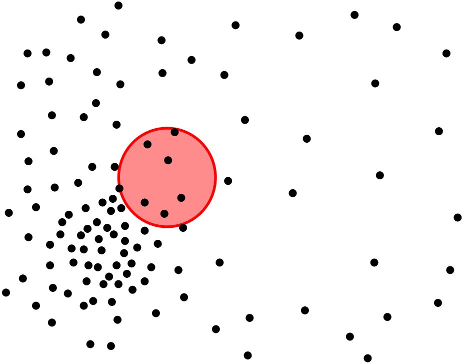
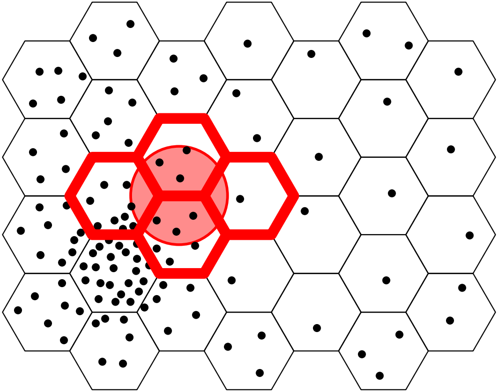


Circular range query: report all points inside query circle Q

Recursive tilings and space-filling curves with little fragmentation

non-recursive tiling

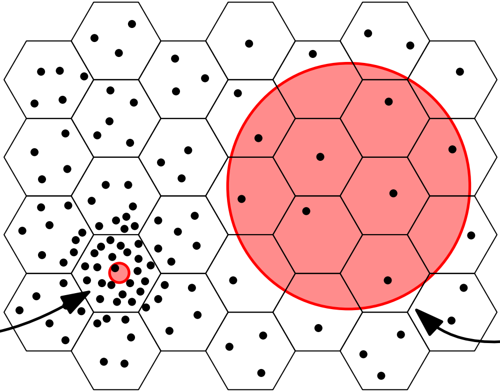


Data structure: store points tile by tile; each tile = contiguous block on disk

Circular range query: report all points inside query circle Q

Query algorithm: retrieve every tile intersecting Q (one disk seek per tile) non-recursive tiling

Small query in crowded tile
→
retrieves many points outside Q from disk



Large query in sparsely populated region
→
many disk seeks for small result

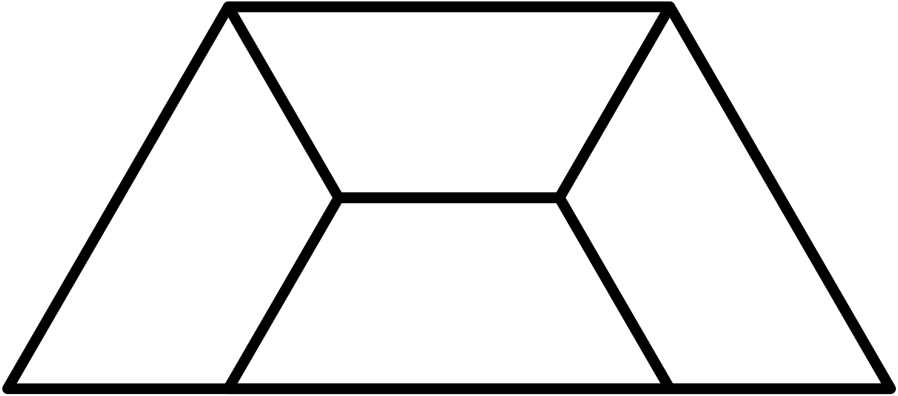
Data structure: store points tile by tile; each tile = contiguous block on disk

Circular range query: report all points inside query circle Q

Query algorithm: retrieve every tile intersecting Q (one disk seek per tile)


recursive tiling

recursive tiling



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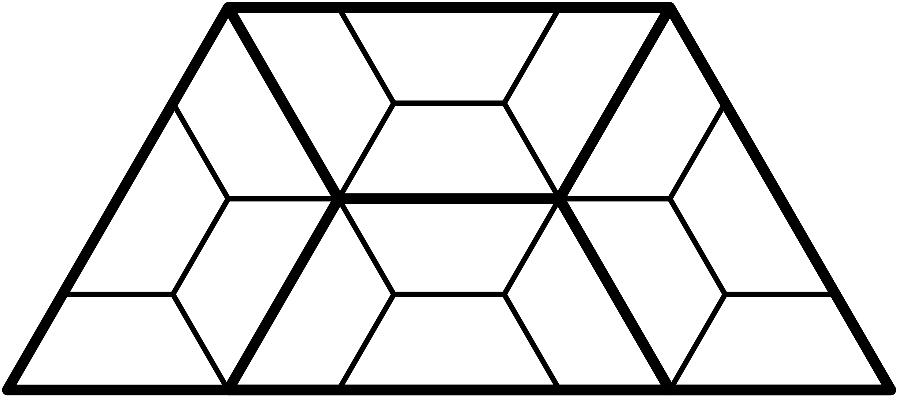

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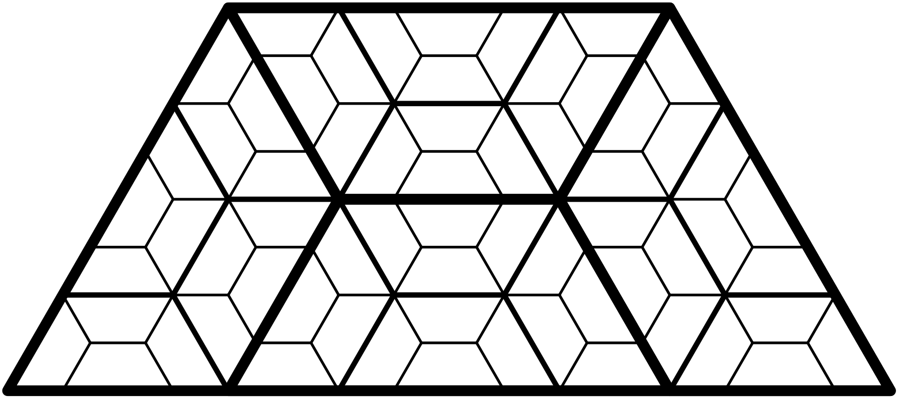

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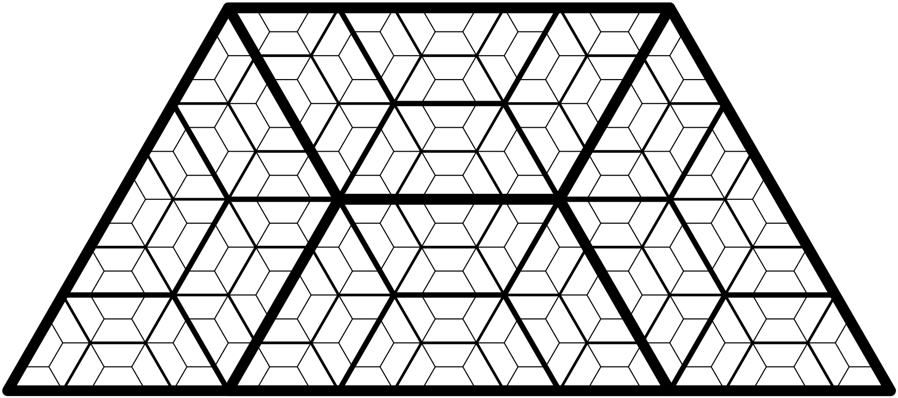

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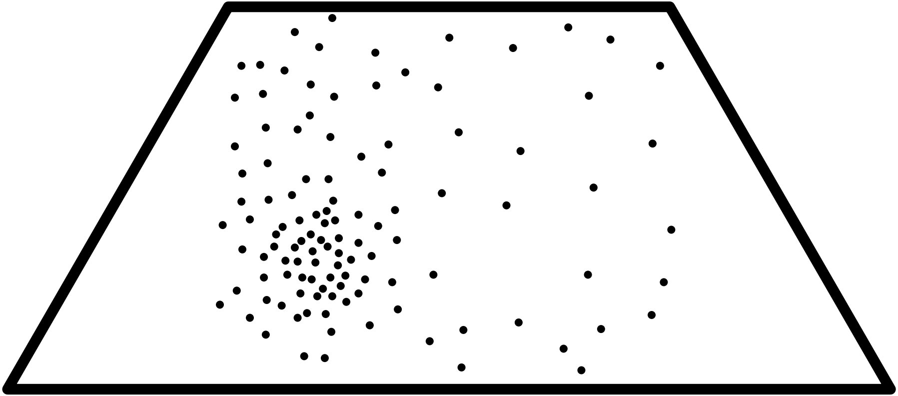

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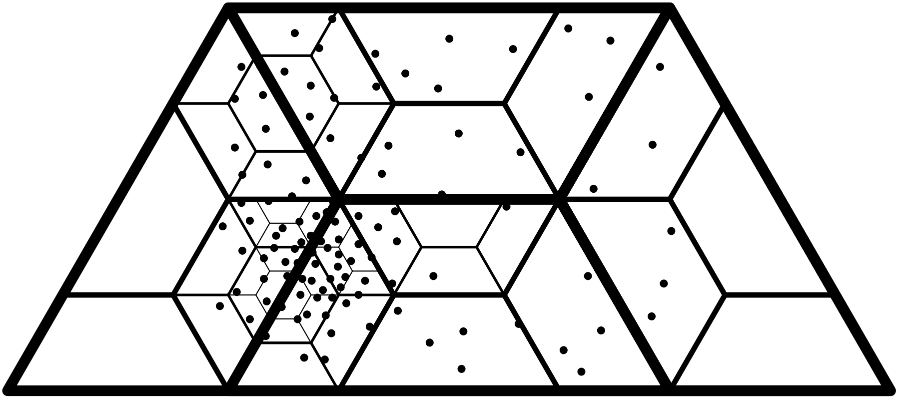
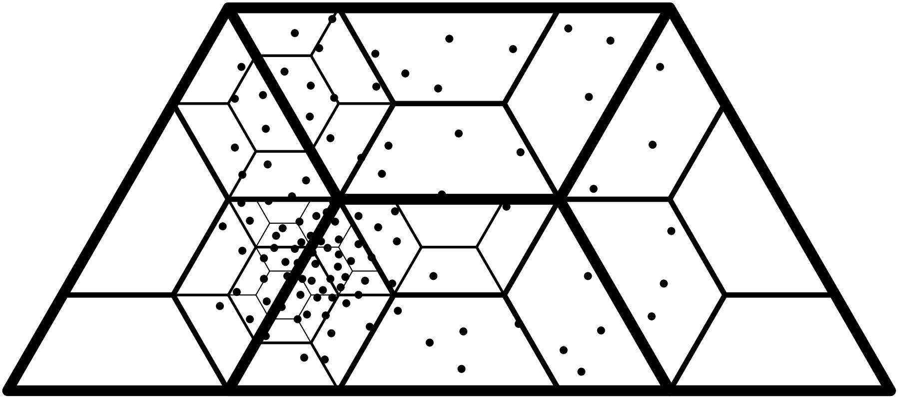


Data structure: store points tile by tile; each tile = contiguous block on disk

(similar to linear quadtree)

Recursive tilings

recursive tiling



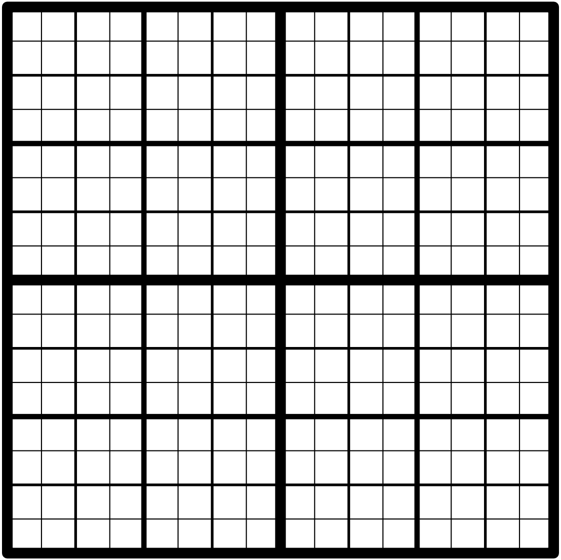
Data structure: store points tile by tile; each tile = contiguous block on disk

Goal 1: a tiles suffice to cover any disk-shaped query range Q *(seek time)*

Goal 2: the tiles that cover Q have total area at most $c \cdot \mathrm{area}(Q)$ *(read time)*

Arrwwid number $=$
smallest a such that there is a constant c such that $\forall Q$ both goals achieved

The Arrwwid number of a tiling with squares

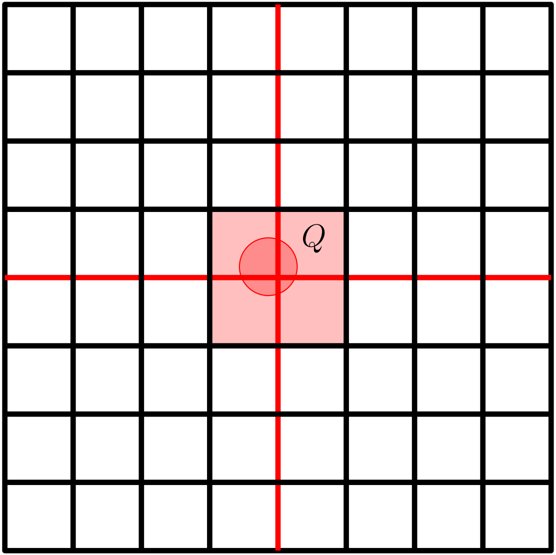


Arrwwid nr. of square tiling ≤ 4:

Arrwwid nr. of square tiling > 3 :

Arrwwid number $=$ smallest a such that $\exists$ constant c such that any disk Q is covered by $\leq a$ tiles of total area $\leq c \cdot \mathrm{area}(Q)$

The Arrwwid number of a tiling with squares



Arrwwid nr. of square tiling ≤ 4:

For disk with radius r, consider grid with tile width $\geq 2r$, $< 4r$:

disk intersected by ≤ 2 grid lines; cover by ≤ 4 tiles meeting there.

Arrwwid nr. of square tiling > 3 :

Arrwwid number $=$ smallest a such that $\exists$ constant c such that any disk Q is covered by $\leq a$ tiles of total area $\leq c \cdot \text{area}(Q)$

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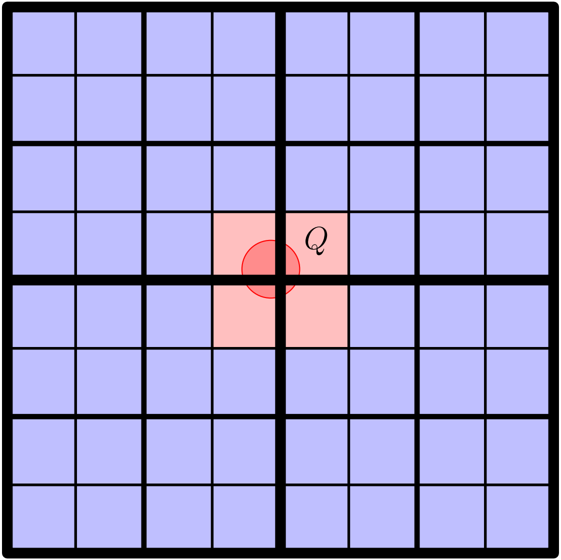

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Arrwwid nr. of square tiling > 3 :

To cover with ≤ 3 tiles,
need common ancestor of red tiles,
can be much bigger than Q
$\rightarrow$ no constant c.

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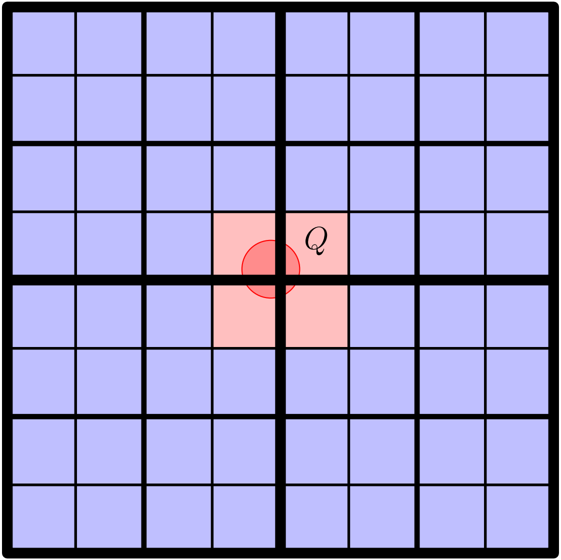

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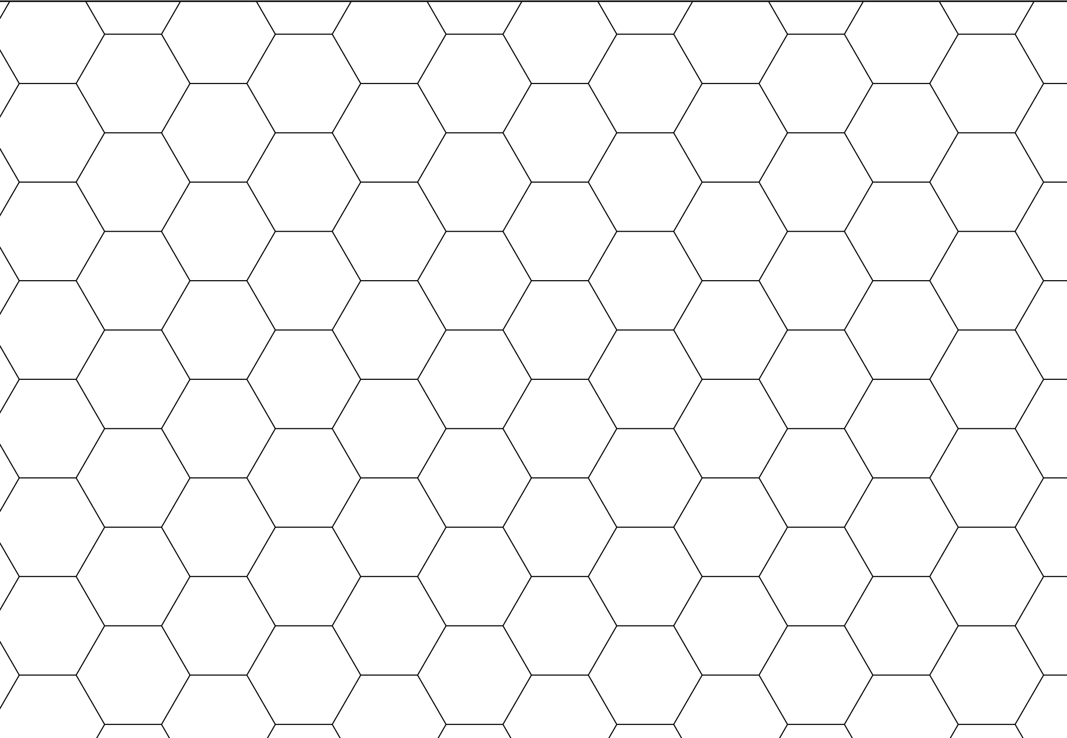
To cover with ≤ 3 tiles, need common ancestor of red tiles, can be much bigger than Q
$\rightarrow$ no constant c.

Arrwwid nr. $\approx$ degree of vertices

Are there recursive tilings with vertex degree 3?

Arrwwid number $=$ smallest a such that $\exists$ constant c such that any disk Q is covered by $\leq a$ tiles of total area $\leq c \cdot \mathrm{area}(Q)$

From non-recursive tilings...



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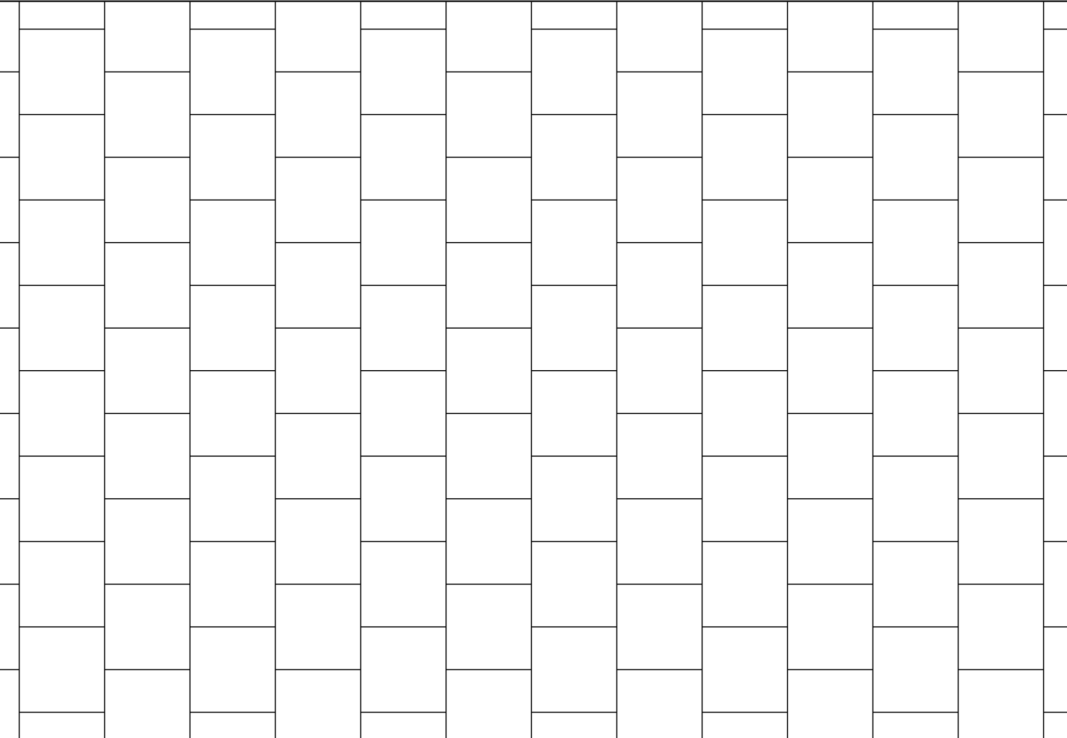
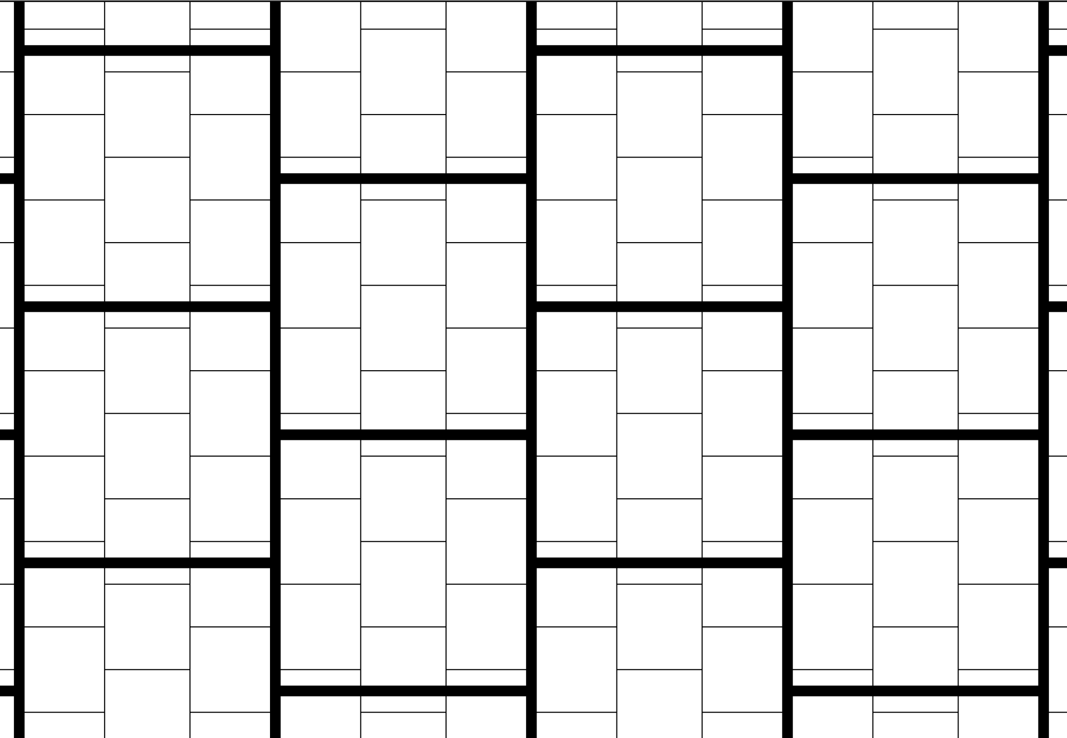


From non-recursive tilings to recursive tilings



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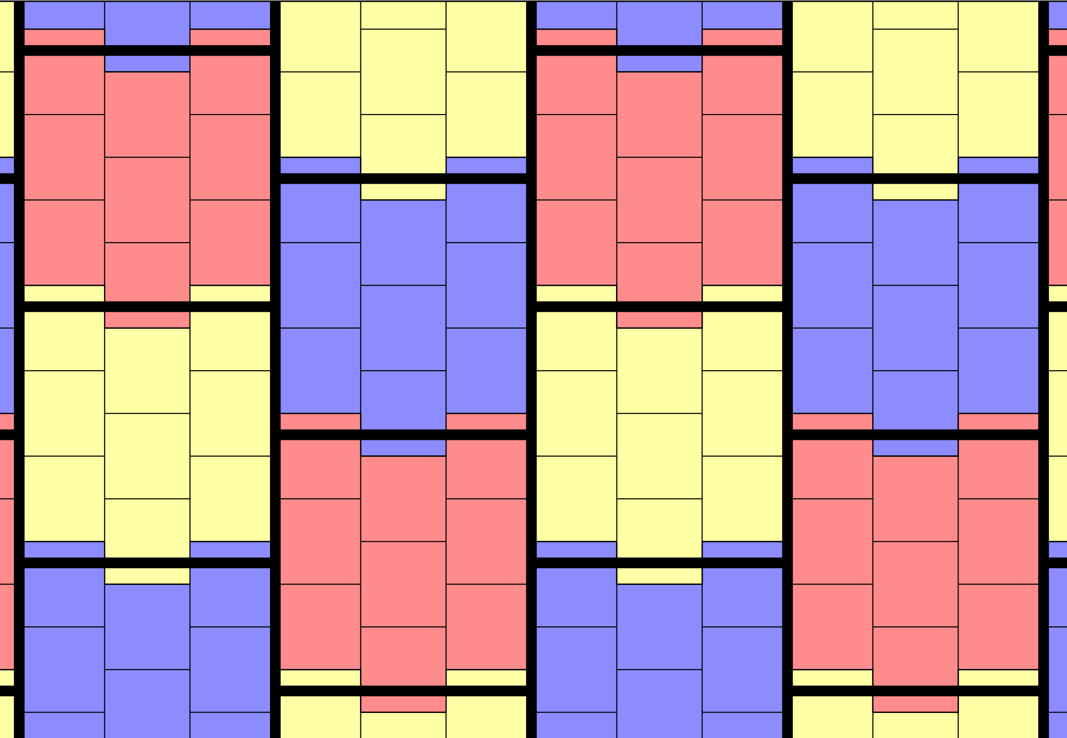

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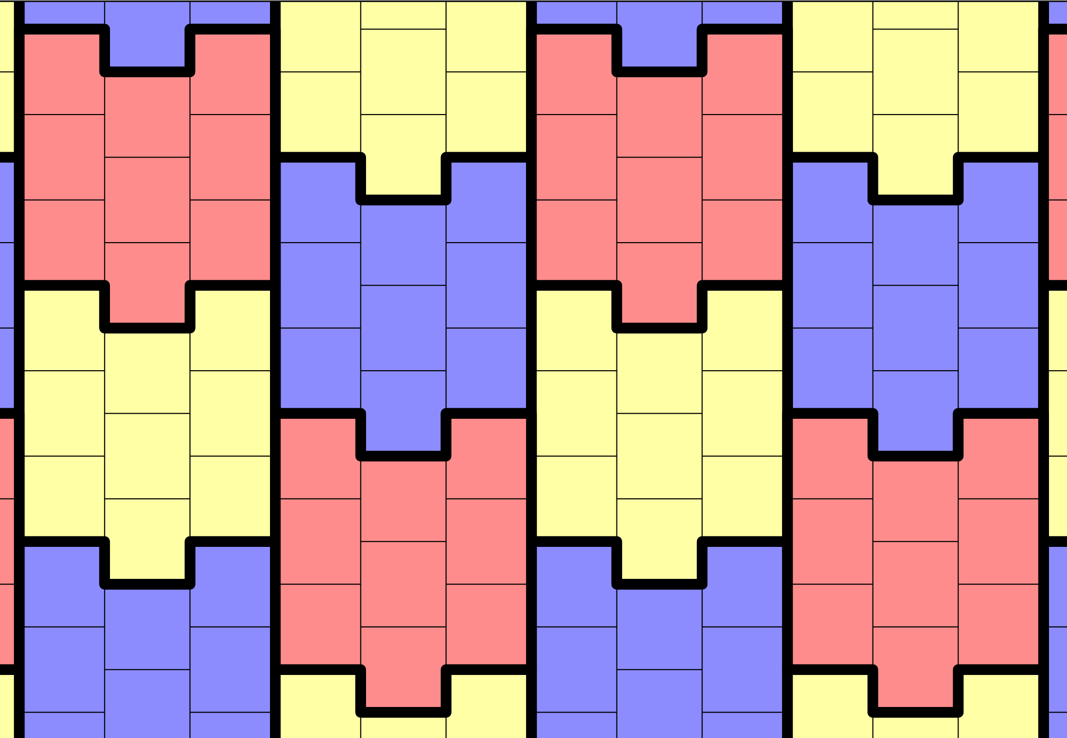

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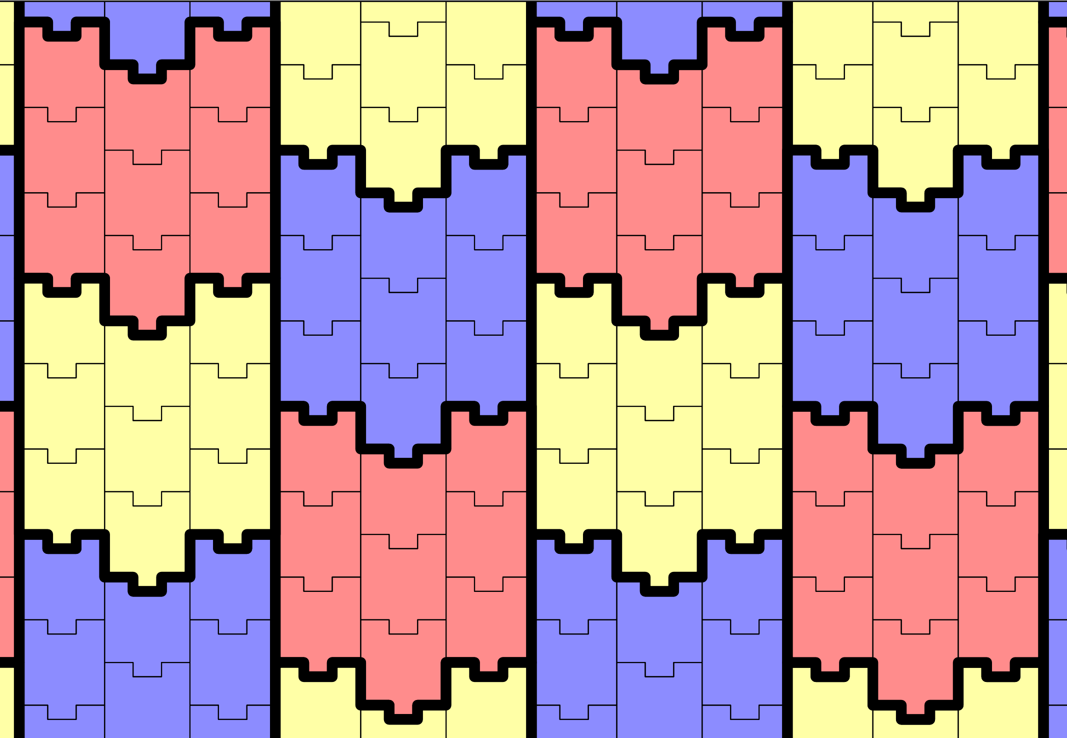

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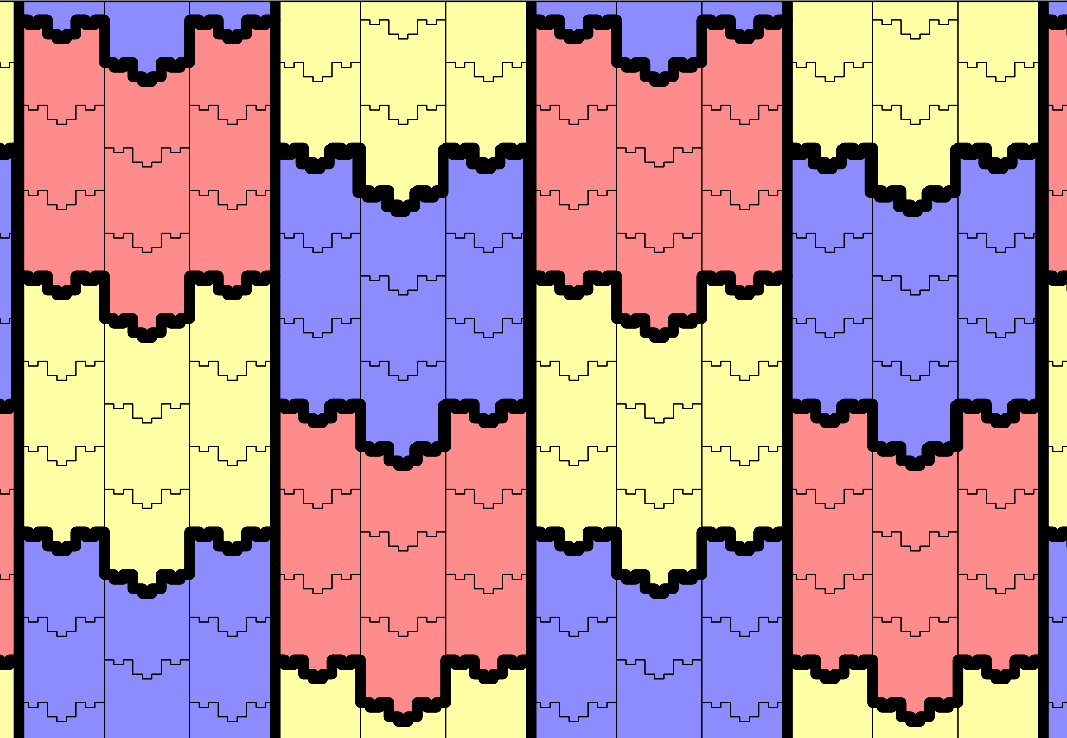

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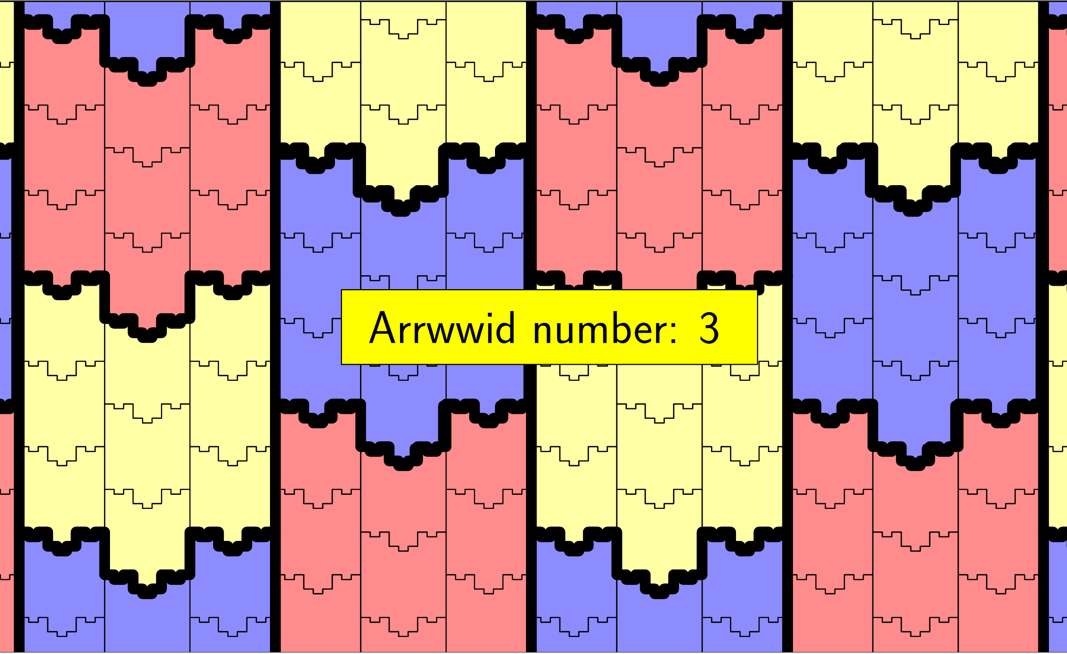

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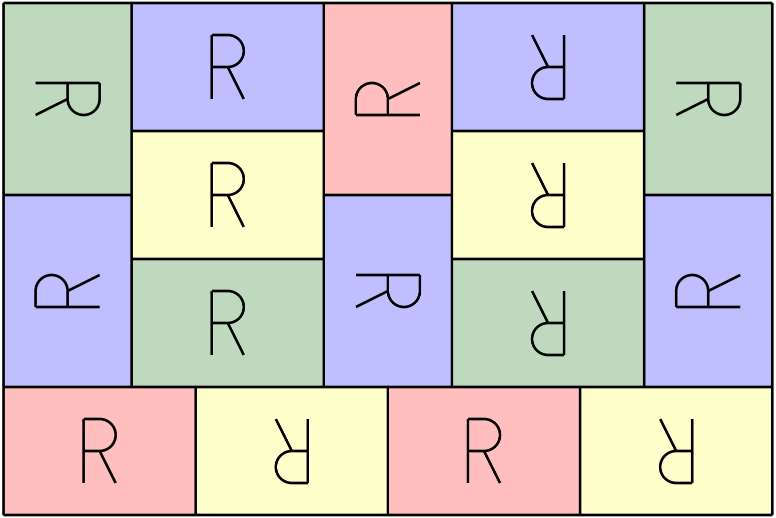
Daun tiling: Arrwwid nr. 3 and "easy" tiles

R:



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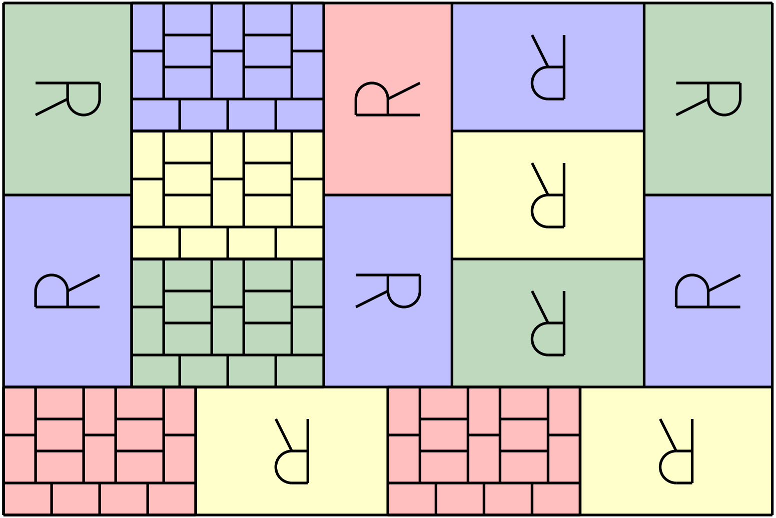

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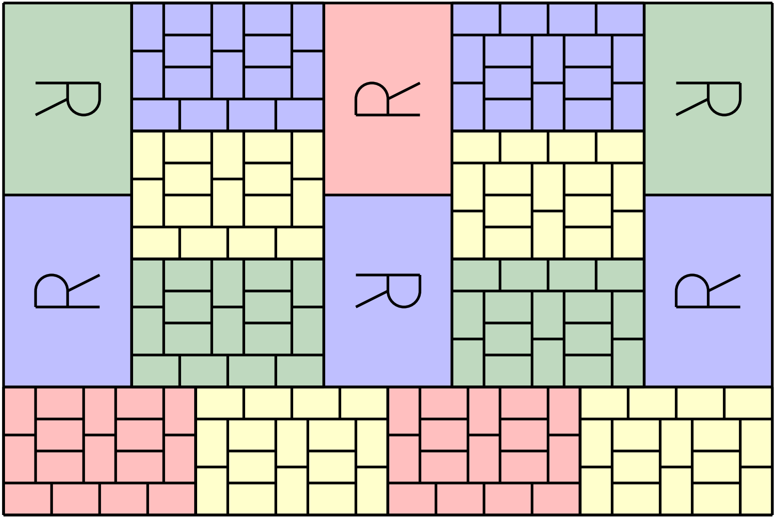

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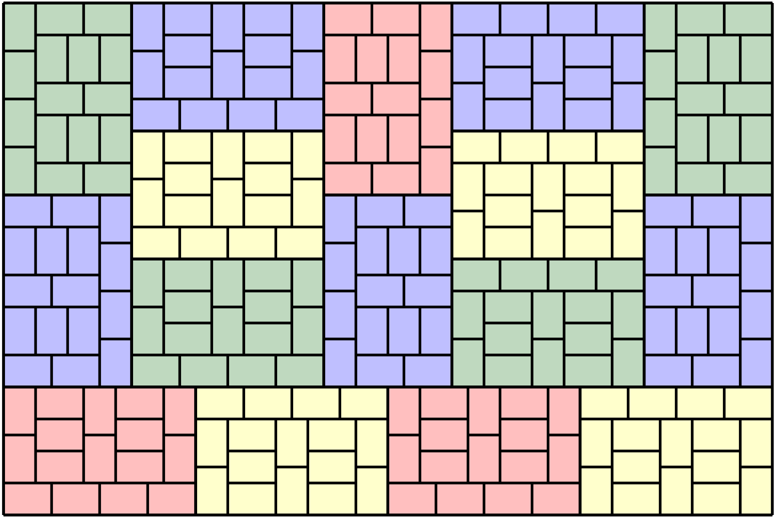

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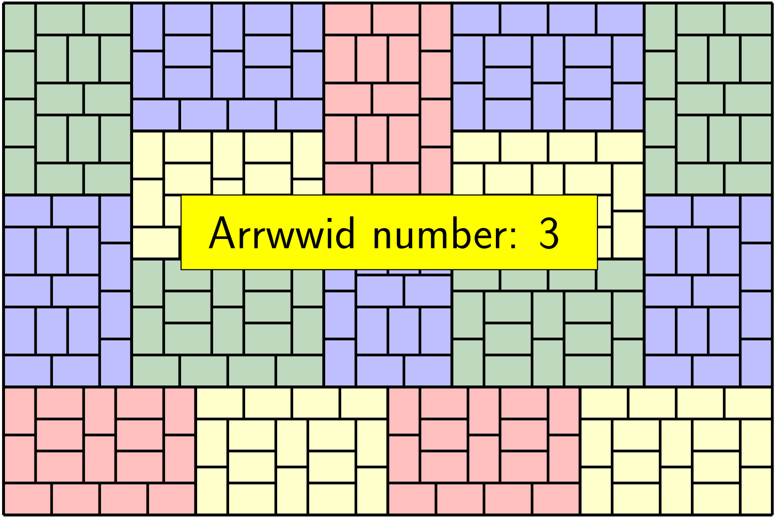

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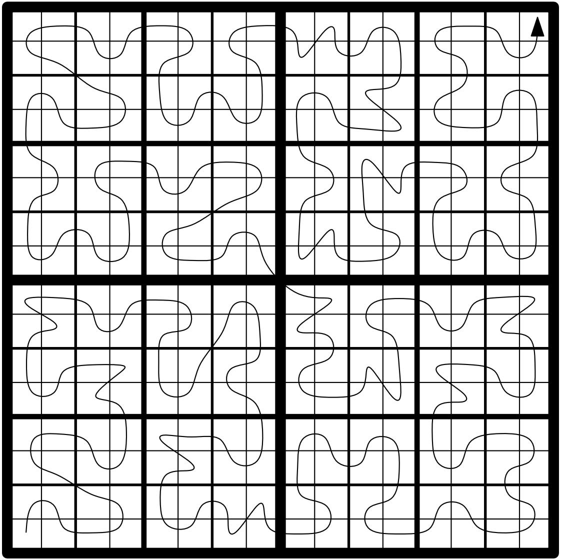
The Arrwwid number of a space-filling curve



space-filling curve =
rec. tiling + ordering rule s.t.

all subtiles within any tile are consecutive

Example: AR^2W^2 order

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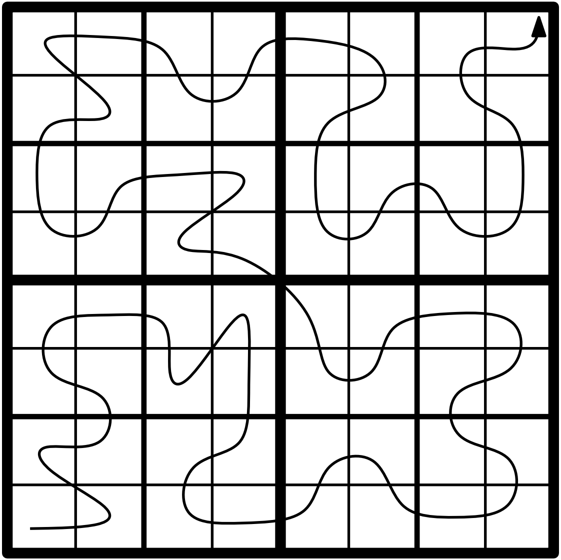

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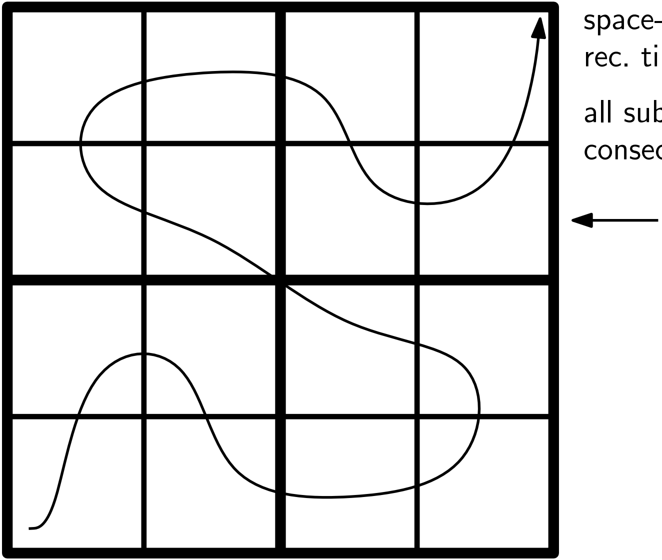

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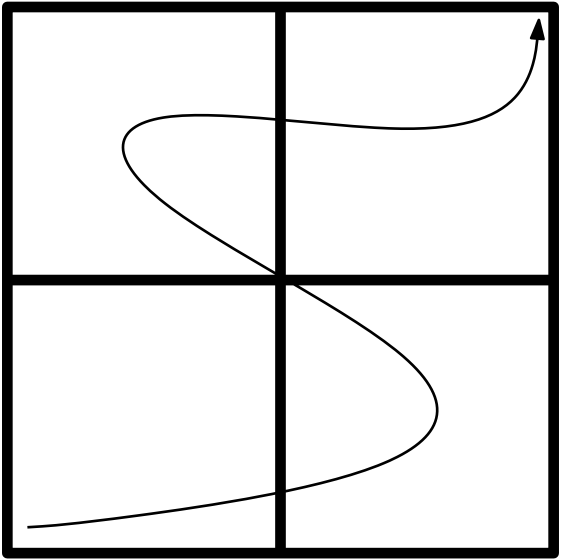

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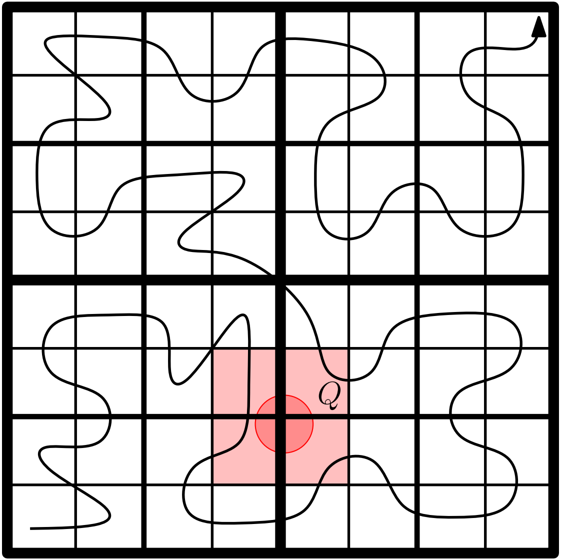

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← Example: AR^2W^2 order

Q covered by 4 tiles;
2 are consecutive in order
→ only 3 seeks

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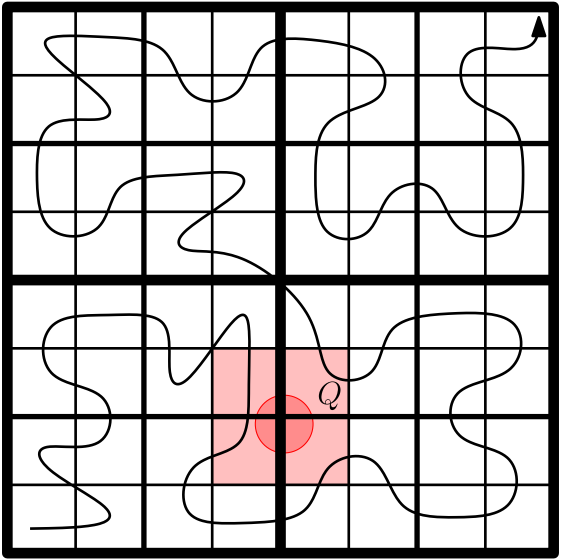

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The Arrwwid number of a space-filling curve

Definition for recursive tilings:

Arrwwid number $=$ smallest a such that $\exists$ constant c such that any disk Q is covered by $\leq a$ tiles of total area $\leq c \cdot \text{area}(Q)$

Definition for space-filling curves (scanning orders of recursive tilings):

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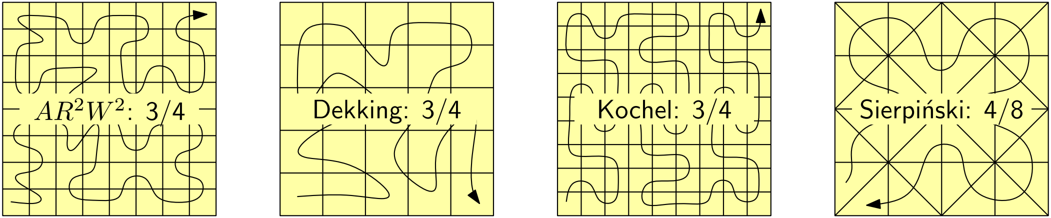
Some curves have smaller Arrwwid nr. than the underlying tiling:



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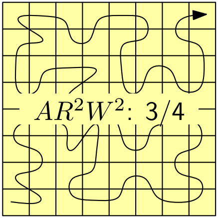

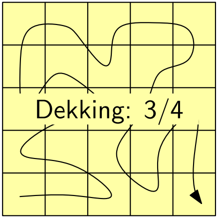

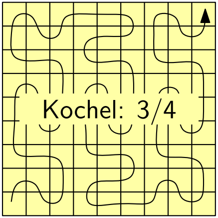

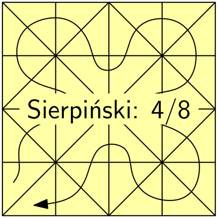

Some tilings have Arrwwid nr. 3. **Some curves have Arrwwid nr. $<$3?**

Asano et al.: Not if tiling divides squares into four squares

Yours truly: **Not ever** (assuming tiles are simply connected)

Results

Herman Haverkort: *Recursive tilings and space-filling curves with little fragmentation*. arXiv:1002.1843 [cs.CG], 2010

best Arrwwid nrs. for tilings	2D
uniform squares	4 [ARRWW]

best nrs. for space-filling curves	2D
uniform squares	3 [ARRWW] ← most known curves have 4

Arrwwid number = smallest a such that $\exists$ constant c such that any disk Q is covered by $\leq a$ (sets of consecutive) tiles of total area $\leq c \cdot \mathrm{area}(Q)$

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best Arrwwid nrs. for tilings	2D
uniform squares	4 [ARRWW]
uniform rectangles	3
uniform tiles of any shape	3

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uniform rectangles	3	
uniform tiles of any shape	→ 3	

lower bound holds for simple tiles

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Results

Herman Haverkort: *Recursive tilings and space-filling curves with little fragmentation*. arXiv:1002.1843 [cs.CG], 2010

best Arrwwid nrs. for tilings	2D	3D	d-D
uniform hypercubes	4 [ARRWW]	8	2^d
uniform hyperboxes	3	6 (lwbd 4)	$\frac{3}{4} \cdot 2^d$ (lwbd:?)
uniform tiles of any shape	3		

best nrs. for space-filling curves	2D
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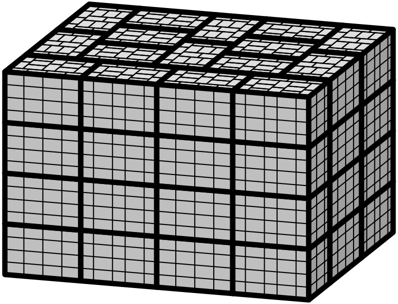
lower bound holds for simple tiles



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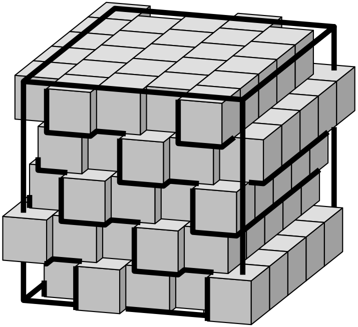

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uniform hypercubes	3 [ARRWW]	8 (lwbd 7)	2^d (lwbd 2^d-1)
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uniform tiles of any shape	3	4	$d+1$ (lwbd:?)

lower bound holds for simple tiles (2D: 3) ...for convex tiles (3D: 4)

Arrwwid number = smallest a such that $\exists$ constant c such that any disk Q is covered by $\leq a$ (sets of consecutive) tiles of total area $\leq c \cdot \text{area}(Q)$

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lower bound holds for simple tiles ...for convex tiles

exponential gap between cubes / general shapes (but maybe also in c)

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THAT'S ALL FOLKS

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