Space-filling curves for 3D mesh traversals

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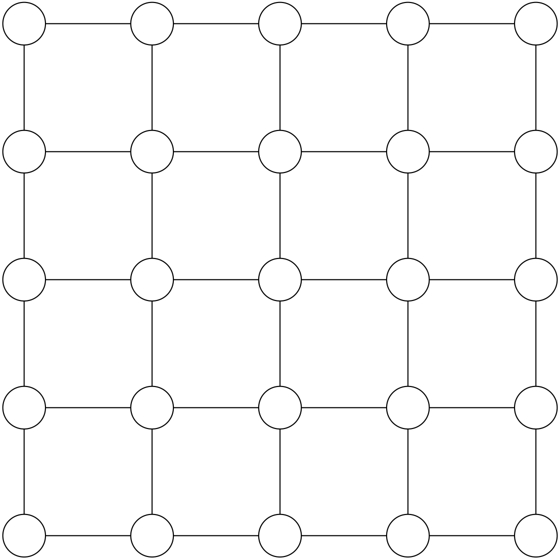


FINITE ELEMENT METHOD:

repeat

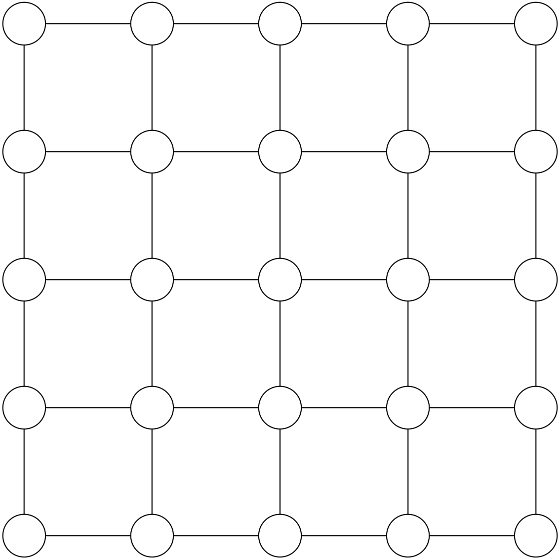
for each square cell

retrieve current values of the four vertices

compute and store new values for the four vertices

until happy

For example: plate with heat sources and sinks at subset of vertices; compute steady-state heat distribution and flow.



In which order? Row by row?

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In what data structure? Matrix?

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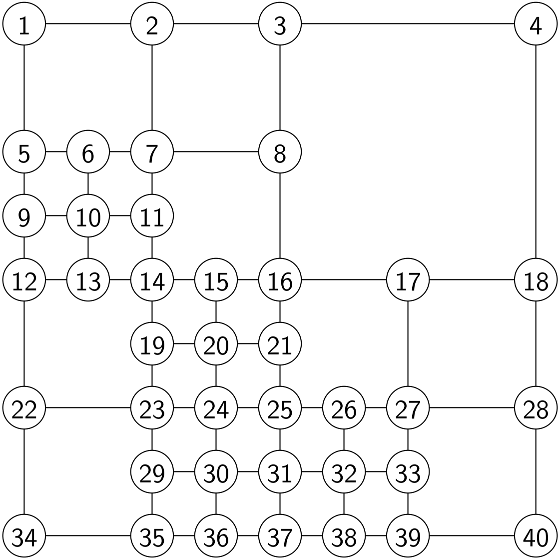
Traversing an irregular grid



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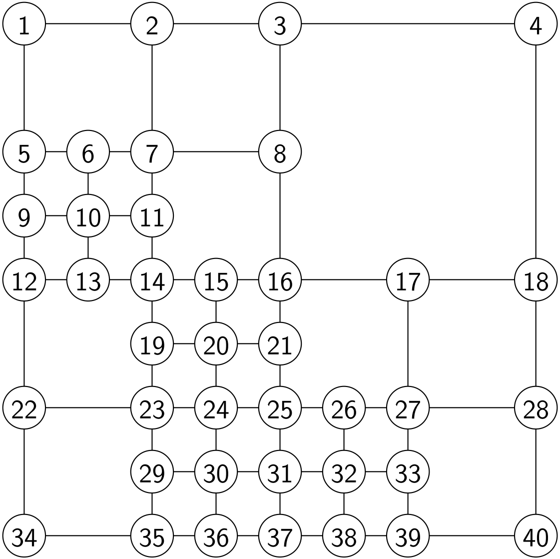

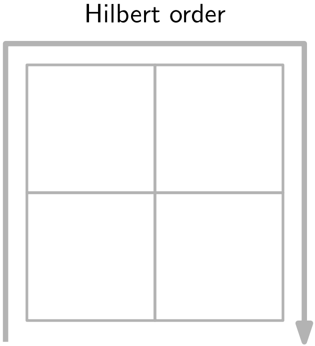

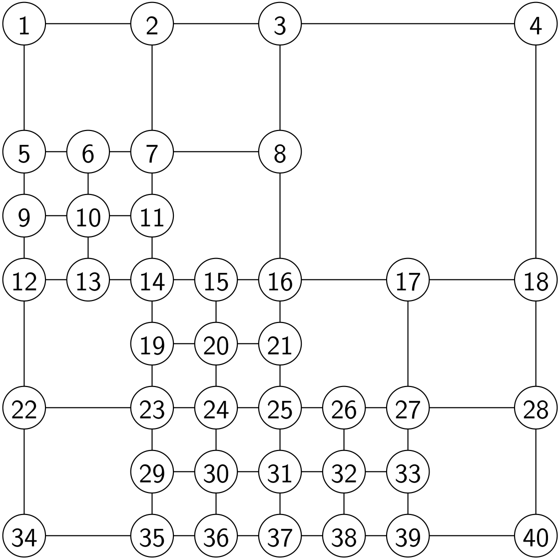
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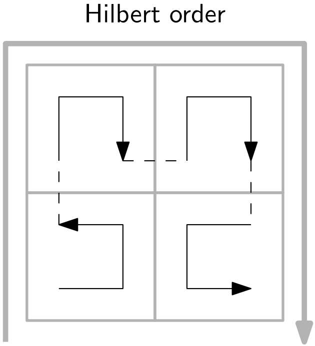
Hilbert order

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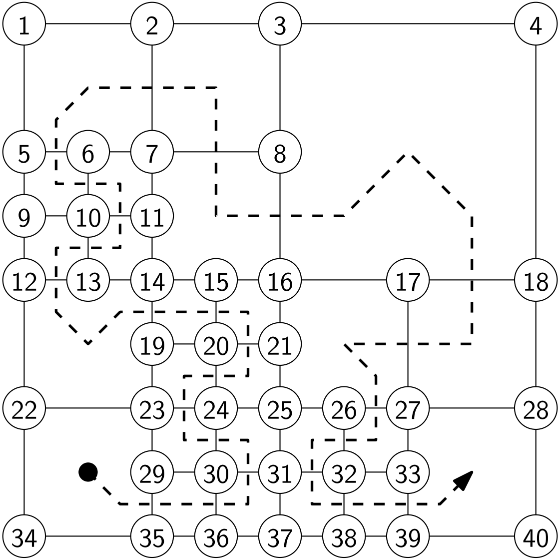

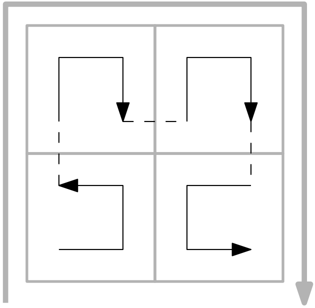

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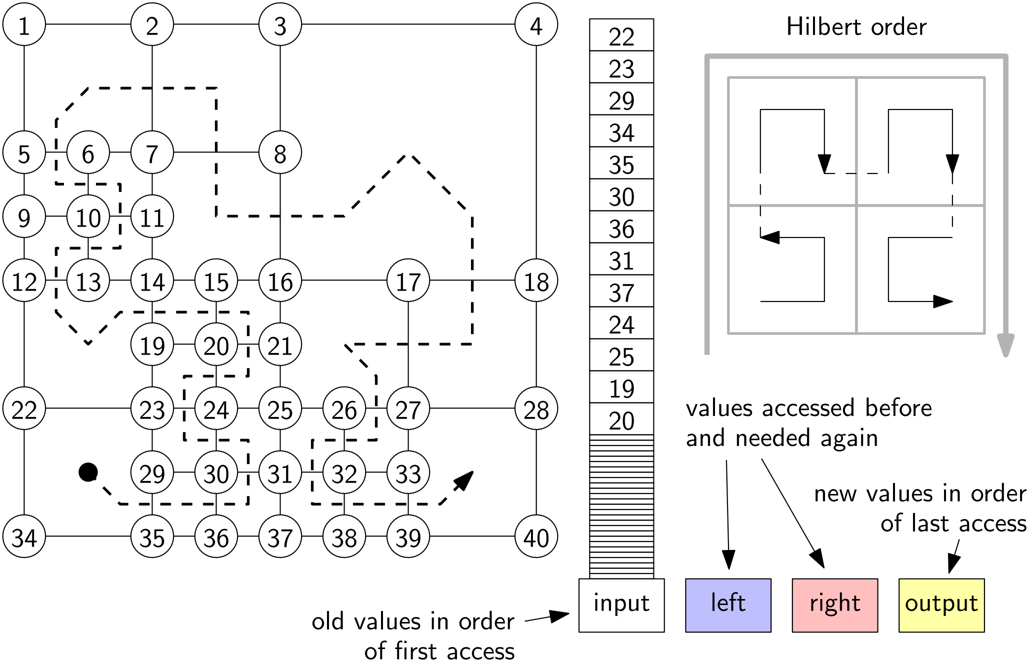

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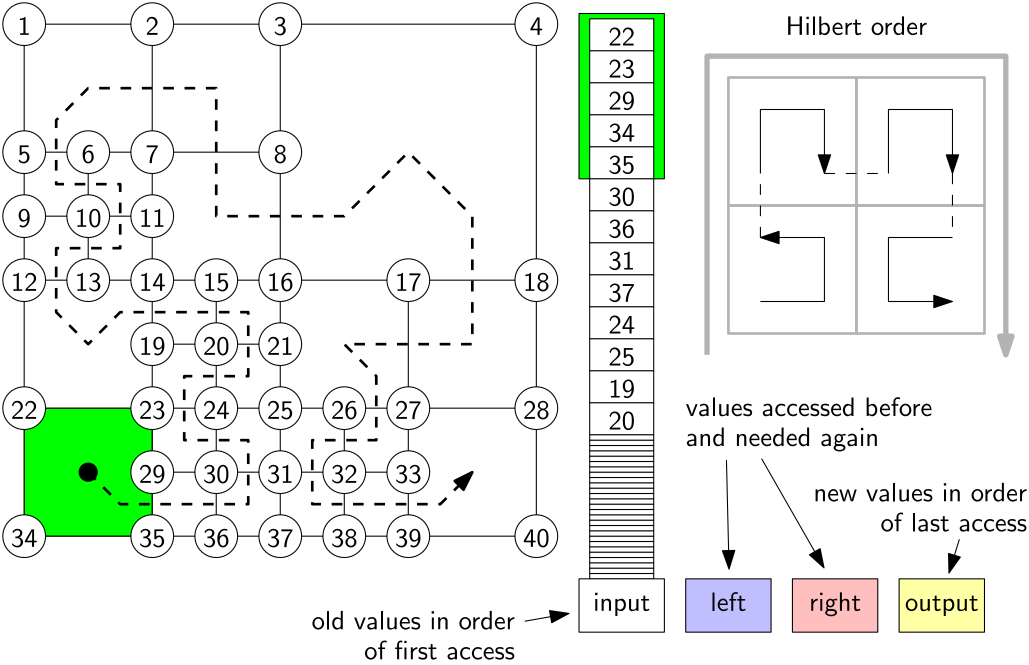

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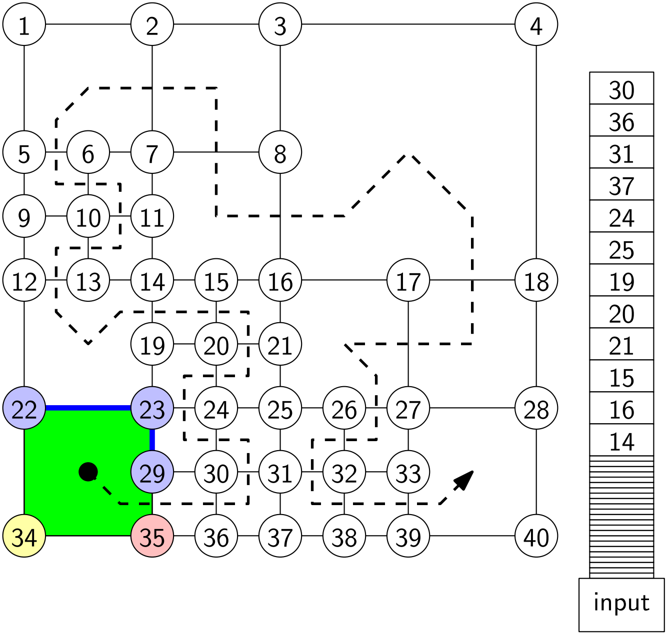

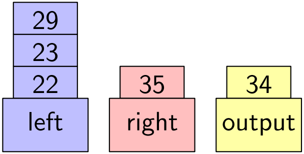

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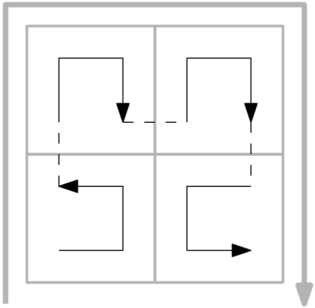

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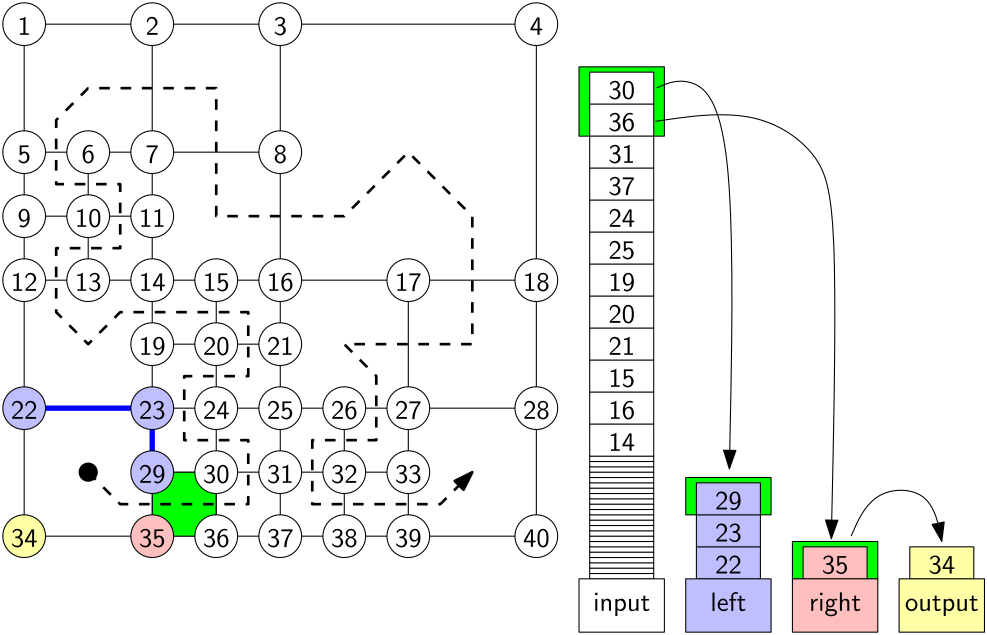

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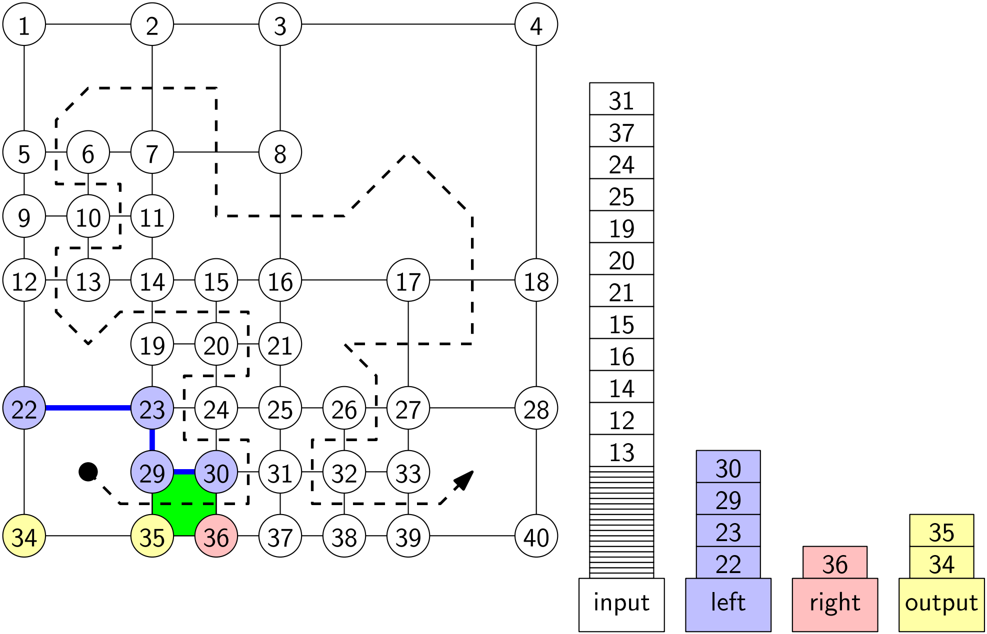

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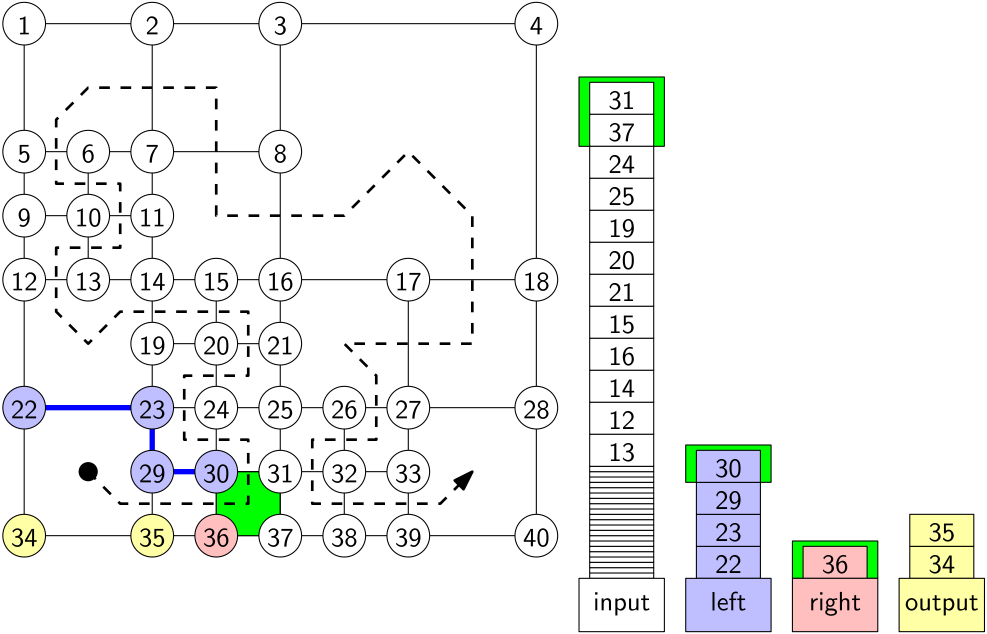

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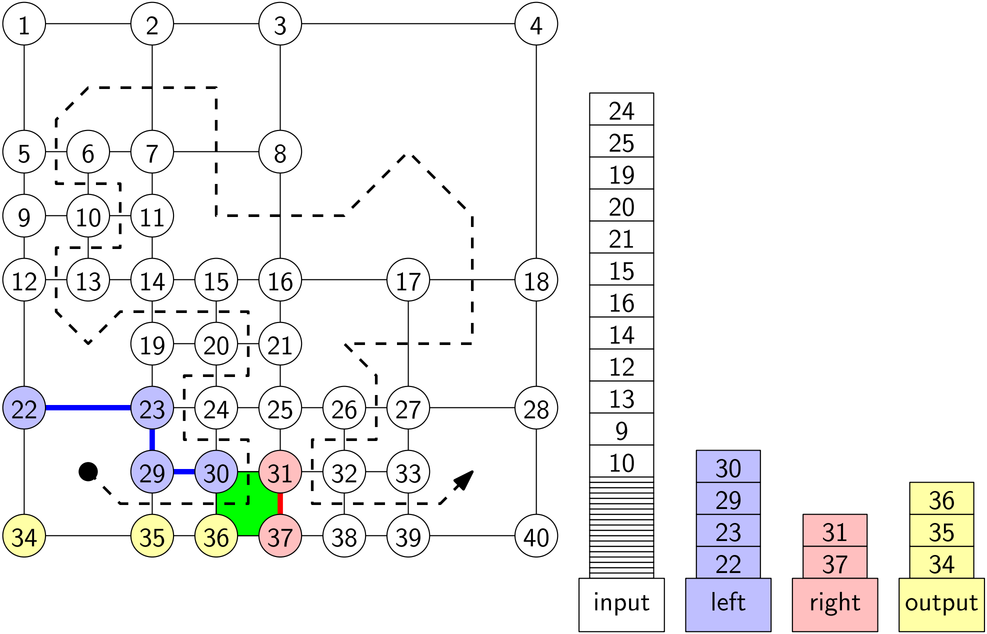

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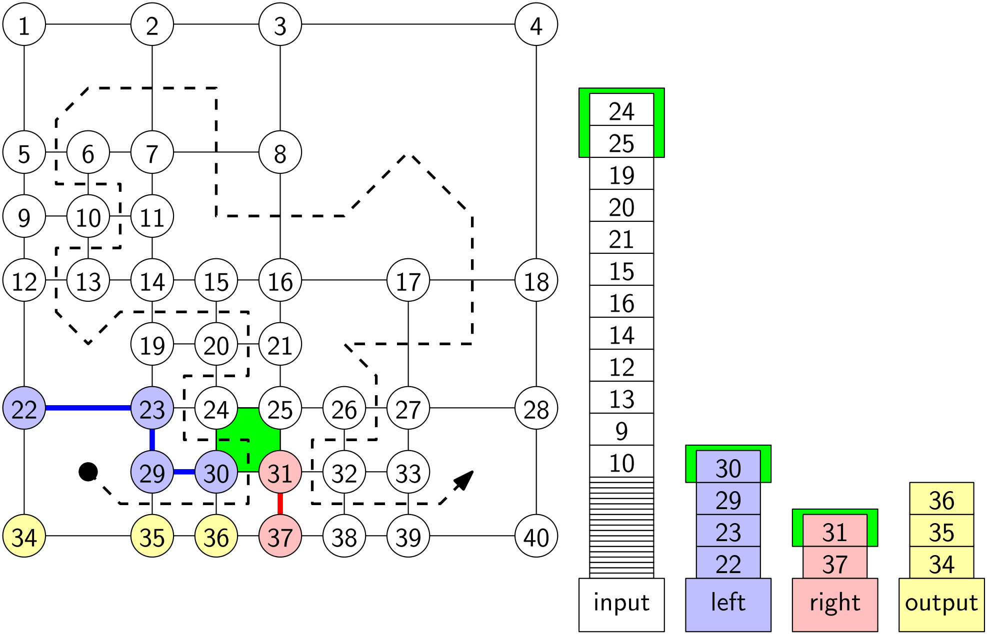

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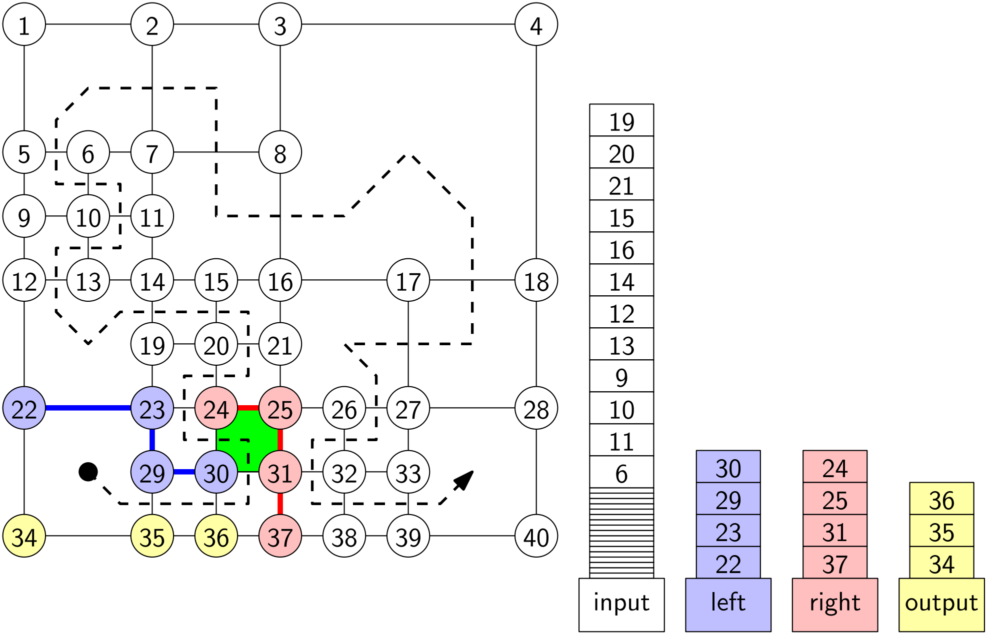

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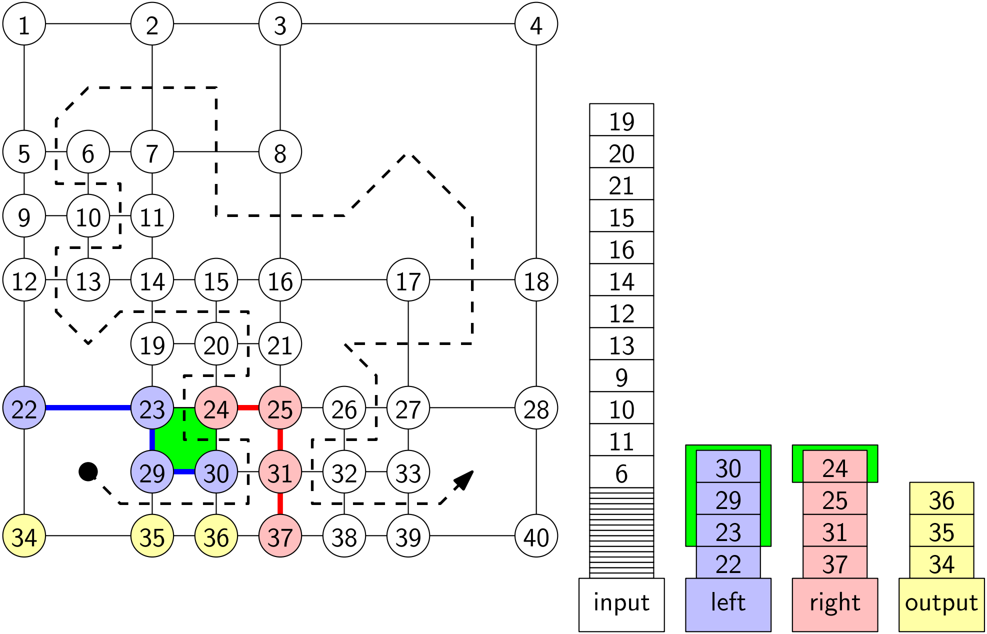

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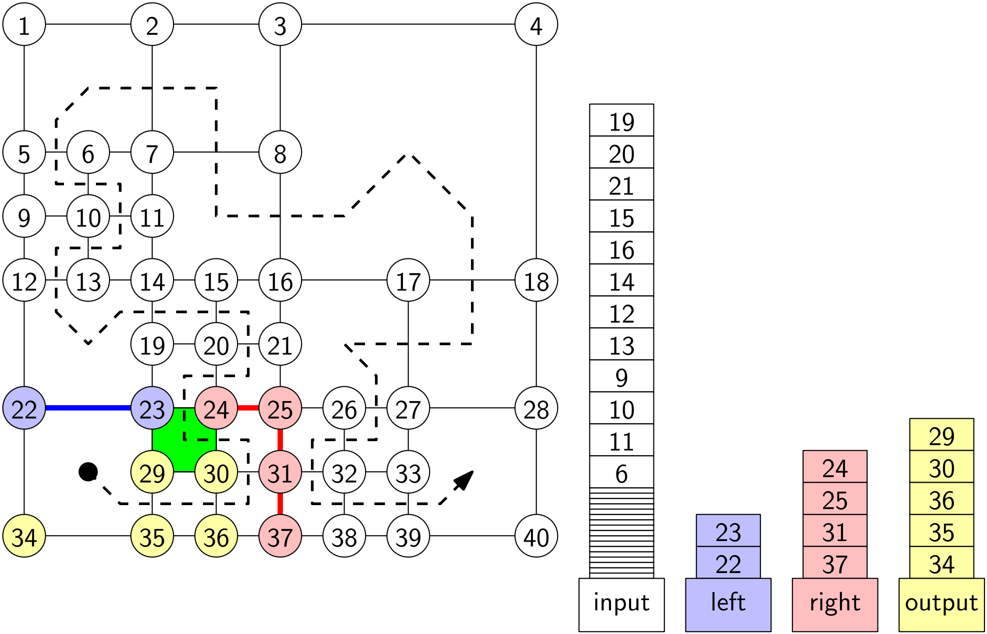

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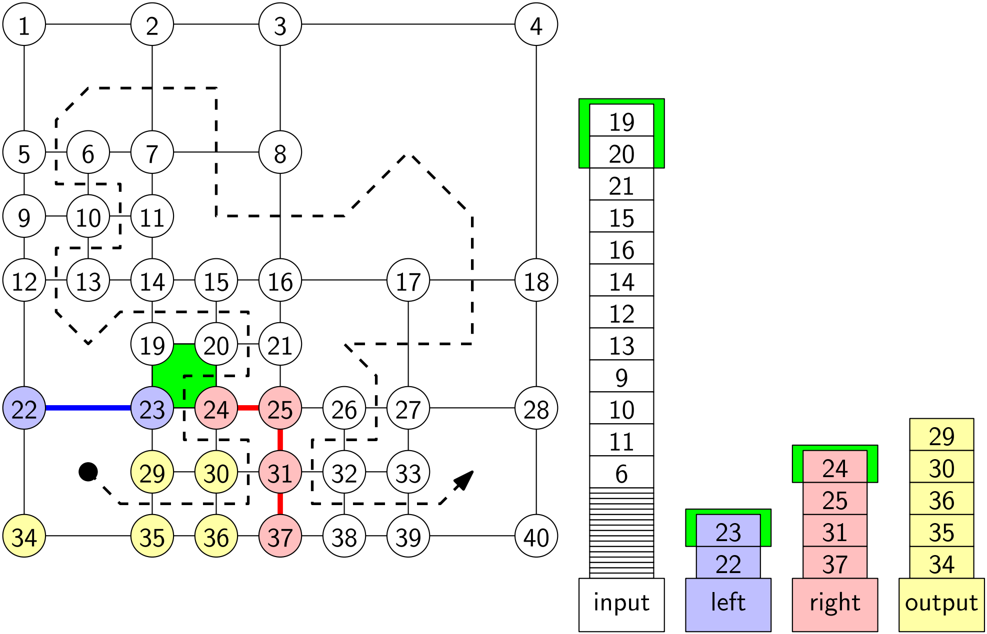

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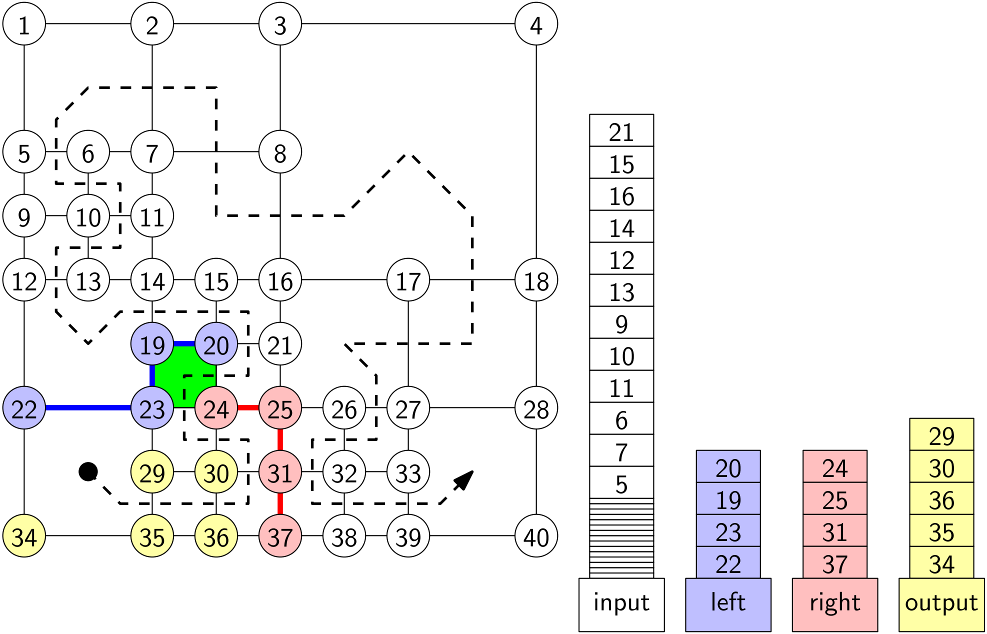

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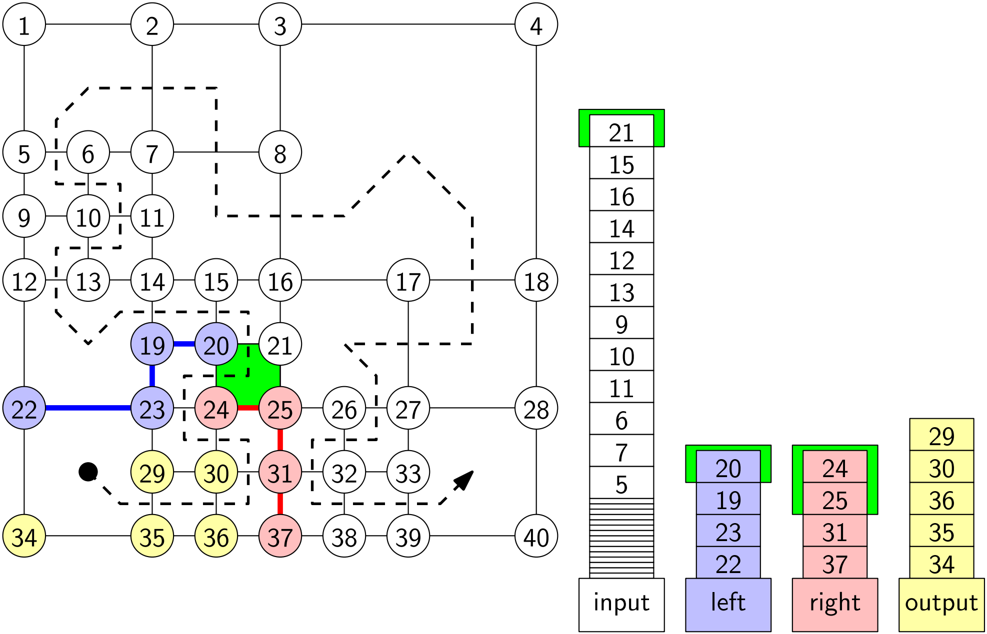

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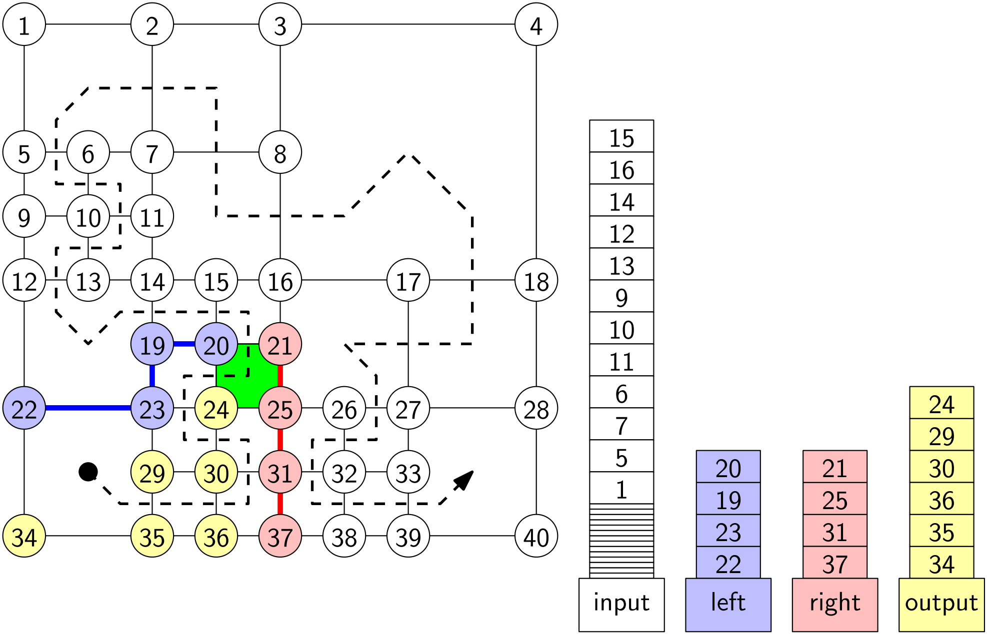

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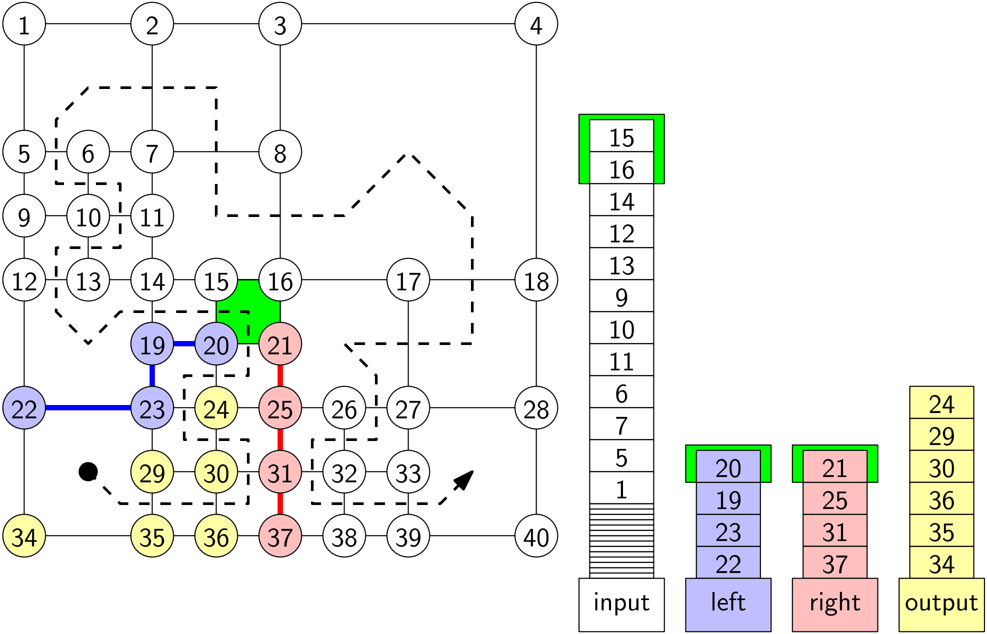

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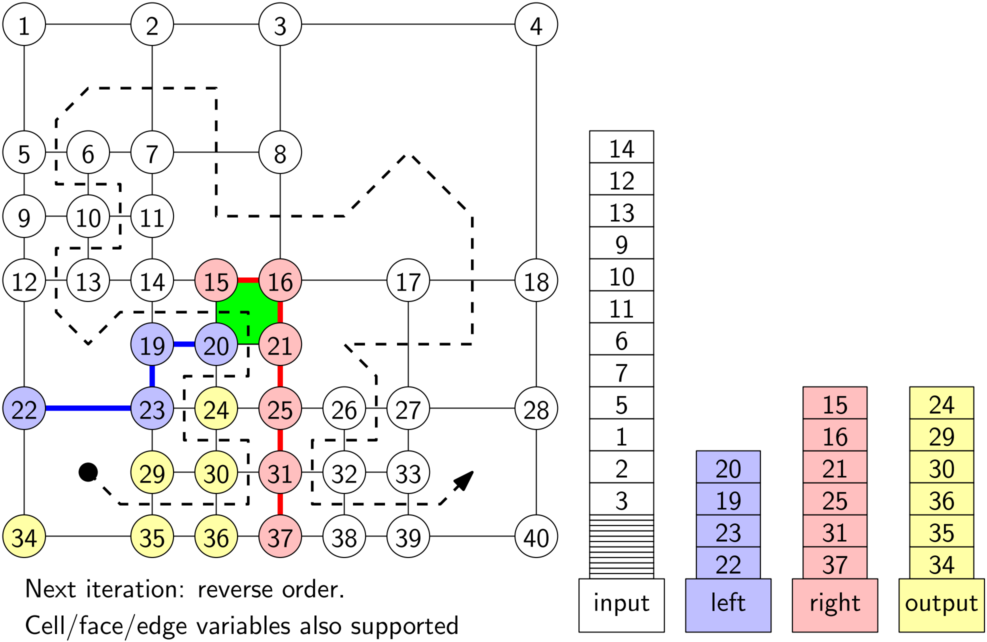
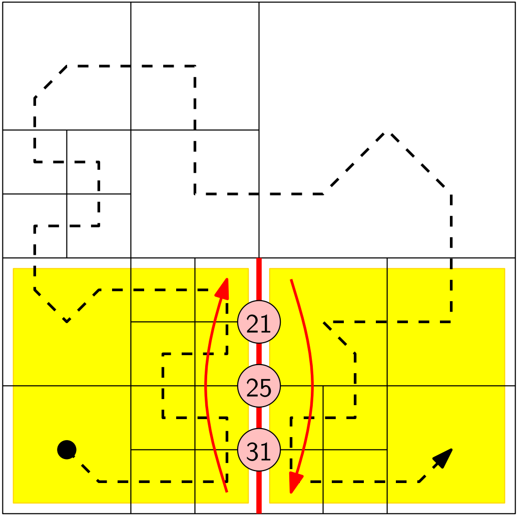
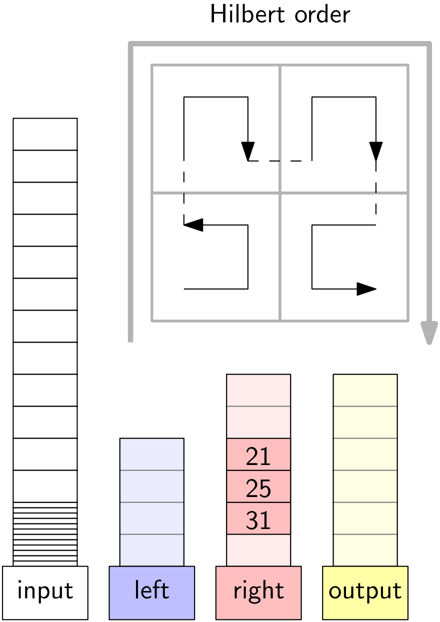


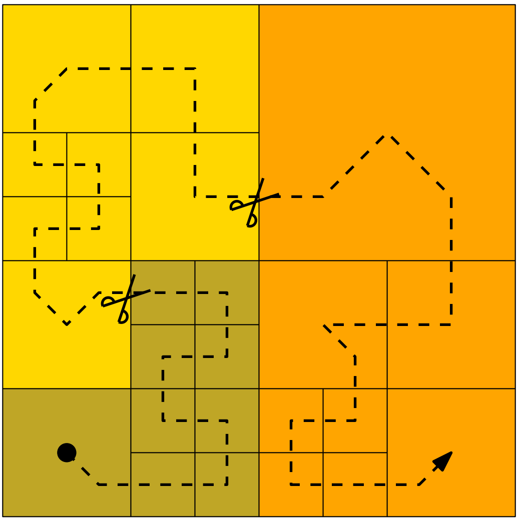
Next iteration: reverse order.

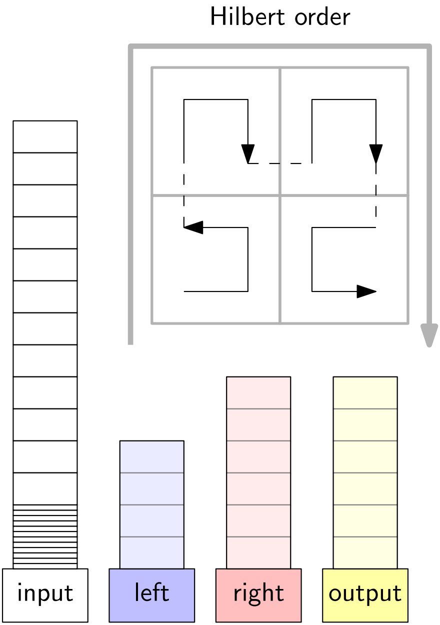
Cell/face/edge variables also supported



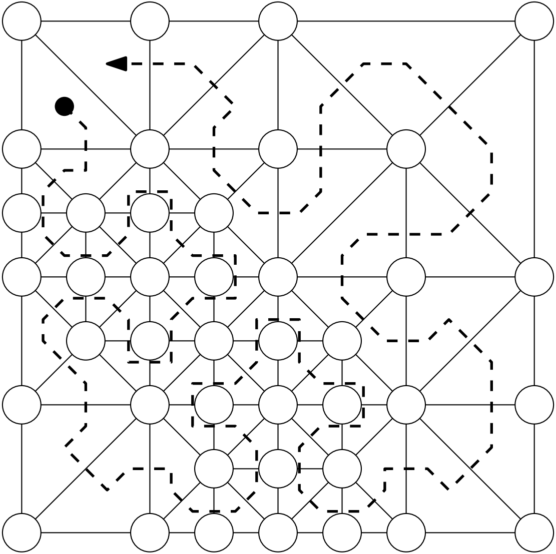


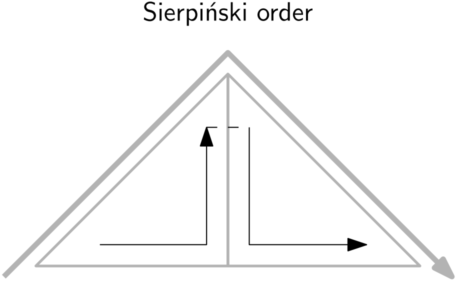
On any edge between two quadrants
order 2nd time = reverse order 1st time



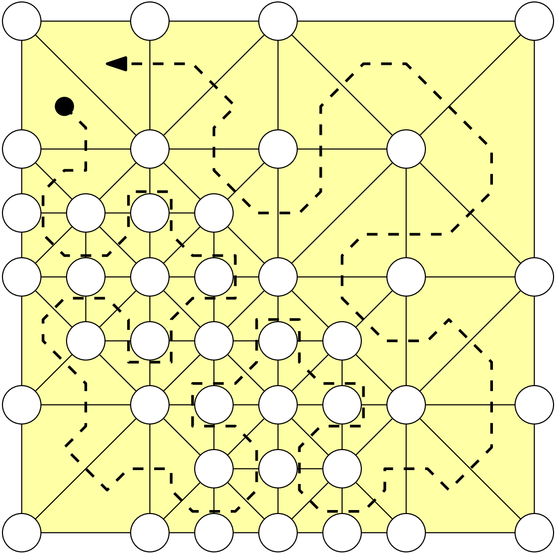


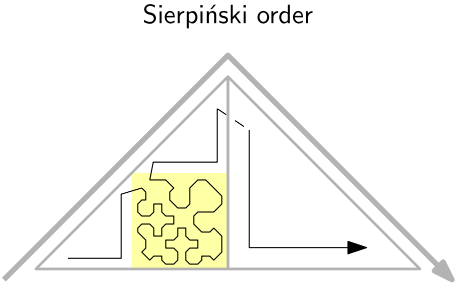
Consecutive squares always share an edge
→(?) well-shaped partitions for load balancing




Sierpiński order

Traversing an irregular grid

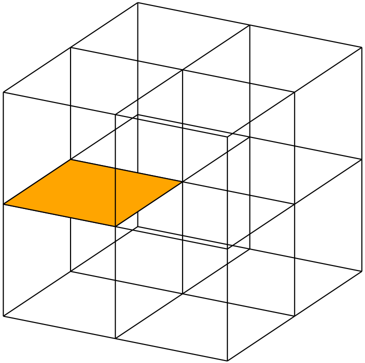




What about 3D?

Desiderata:

- *octree traversal* (cubes/simplices recursively subdivided into 8 parts)
- *face-continuous*: consecutive cells share a face
- *palindromic*: for each pair of adjacent cubes/simplices C_1 and C_2,
  second traversal of common face is reverse of first traversal



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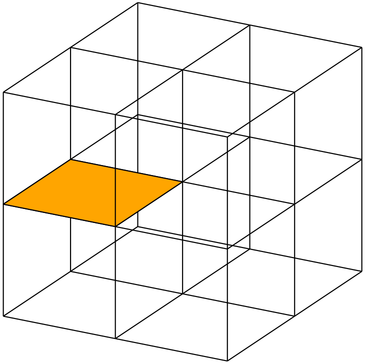

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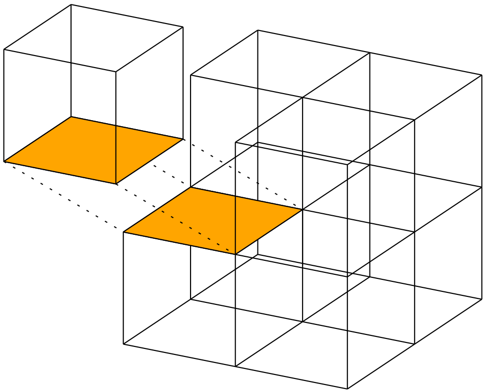

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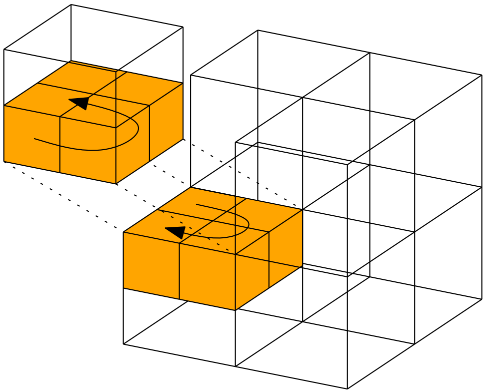

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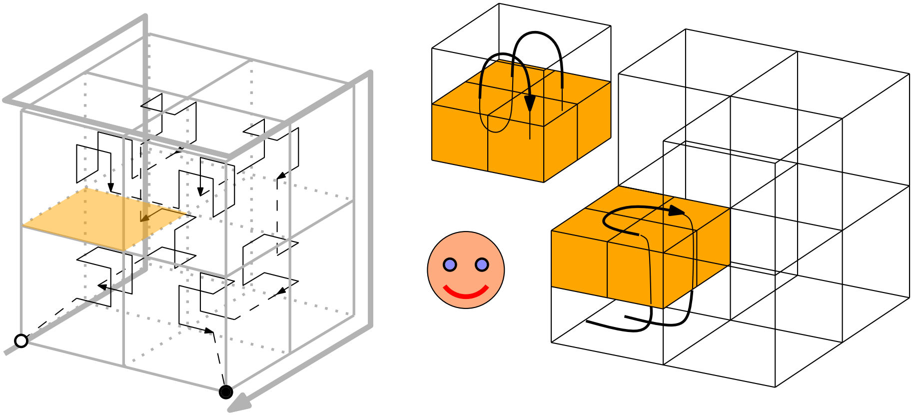

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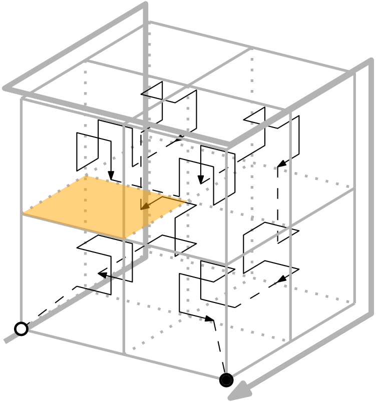

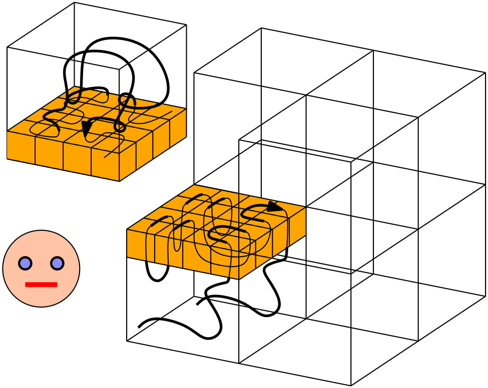

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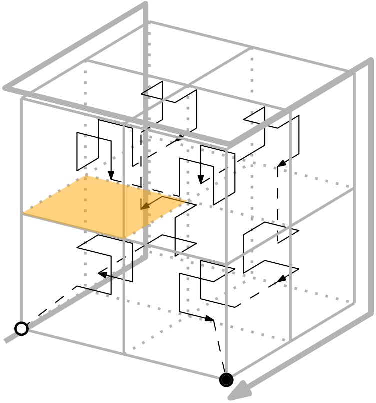

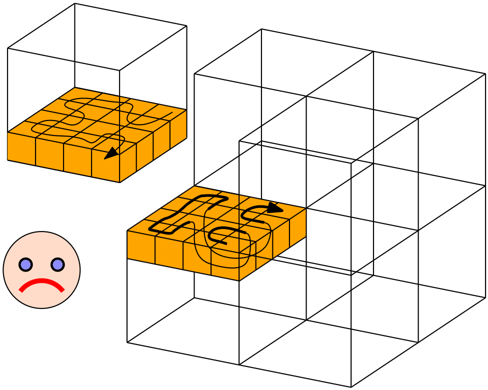

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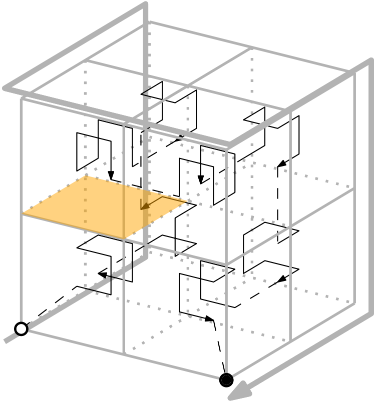
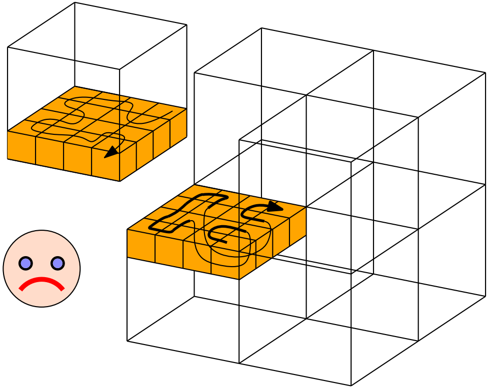




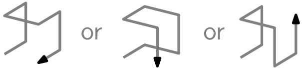
Brute-force search $4 \times 4 \times 4$ finds solutions, but all self-similar recursive expansions fail.

Brute-force search $8 \times 8 \times 8$ is infeasible (and would still generate false positives).

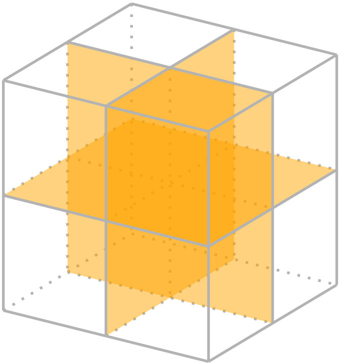
What about cubes?

Generate $S =$ all $4 \times 4 \times 4$ traversals that:

- traverse octants one by one;



- are face-continuous;
- have matching patterns on opposite sides of each of the **twelve interior 2×2-faces**.



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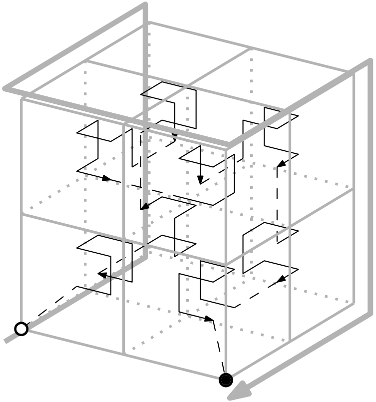
Finds 8384, for example:



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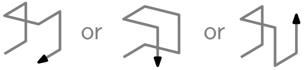

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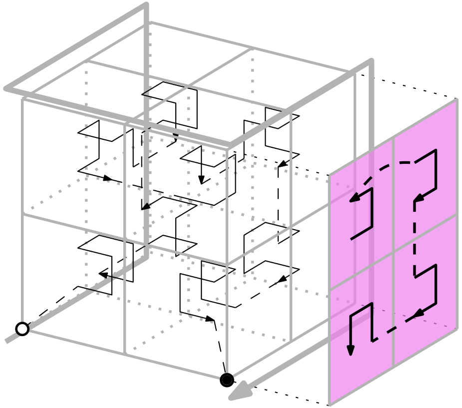

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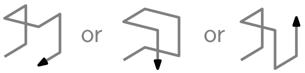

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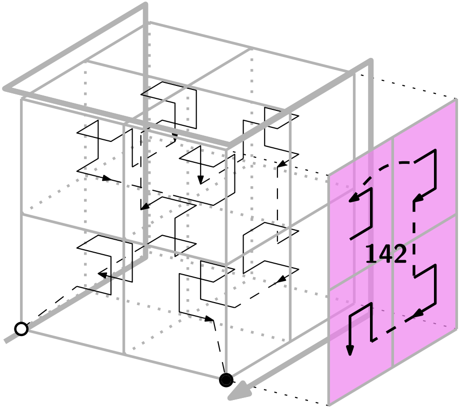

Identify face patterns by id $\in \{0, ..., 255\}$

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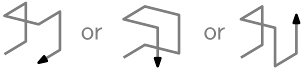

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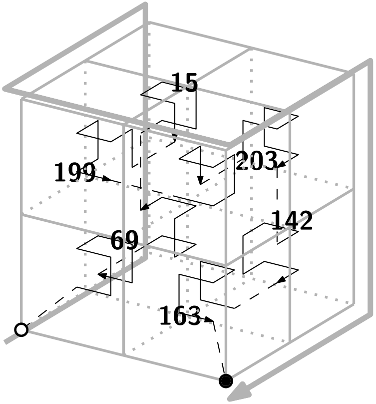

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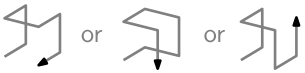

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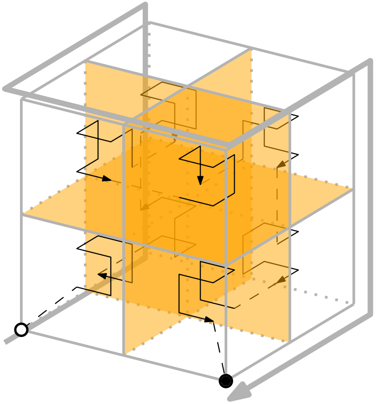

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$48 \times 8384 = 402\,432$

traversals obtained by reflections and rotations

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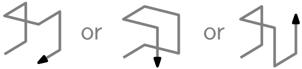

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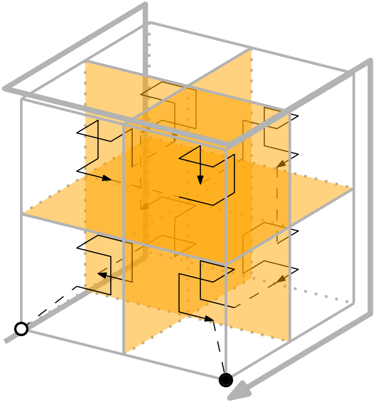

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traversals obtained by reflections and rotations

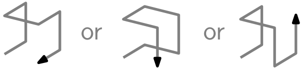
Identify face patterns by id $\in \{0, \ldots, 255\}$

Try to refine $4 \times 4 \times 4$ traversals from S into $8 \times 8 \times 8$ traversals by replacing $2 \times 2 \times 2$ traversals in octants by $4 \times 4 \times 4$ traversals from S'

What about cubes?

Generate S = all $4 \times 4 \times 4$ traversals that:

- traverse octants one by one;
- are face-continuous;
- have matching patterns on opposite sides of each of the **twelve interior 2×2-faces**.

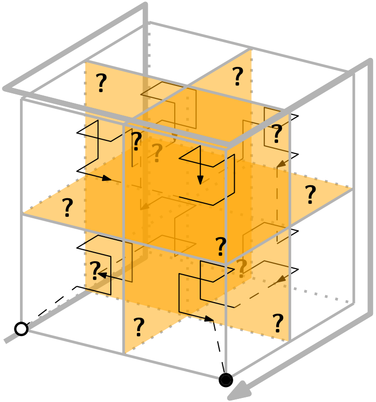


Finds 8384, for example:

$S' =$

$48 \times 8384 = 402\,432$

traversals obtained by reflections and rotations



Identify face patterns by id $\in \{0, ..., 255\}$

For each traversal from S:

exhaustive search of all choices of patterns $\in \{0, ..., 255\}$ for 12 interior faces:

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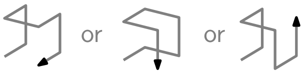

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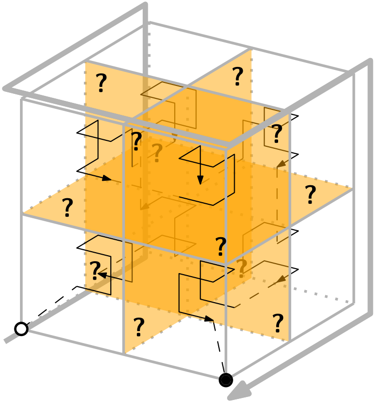

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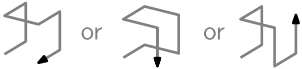

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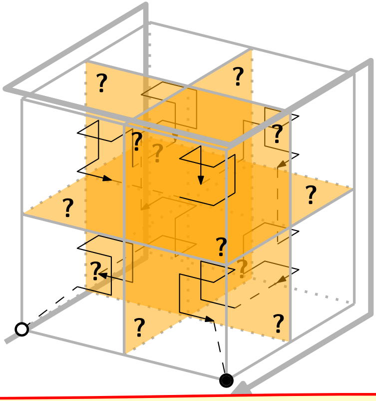

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traversals obtained by reflections and rotations

Identify face patterns by id $\in \{0, ..., 255\}$

unsuccessful for all but 410 traversals from S
(no $8 \times 8 \times 8$-refinement exists)

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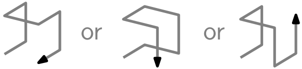
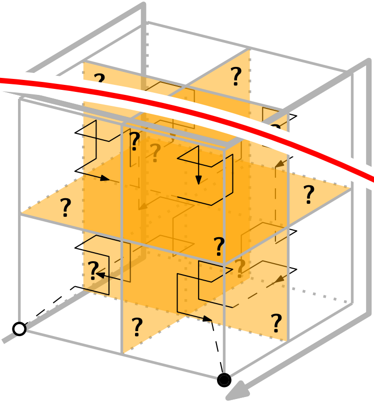


Finds ~~8384~~ **410**, for example:

S' = **410** **19 680**

48 × ~~8384~~ = ~~402 432~~

traversals obtained by reflections and rotations



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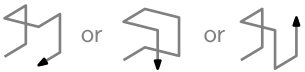

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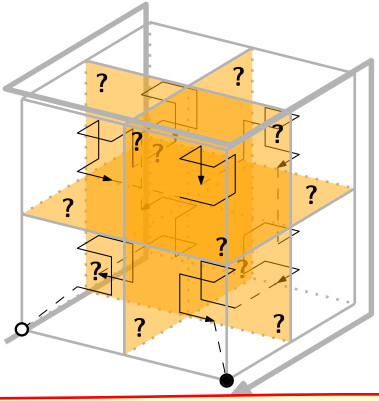

$S' =$ **410** **19 680**
$48 \times \cancel{8384} = \cancel{402\,432}$
traversals obtained by reflections and rotations

Identify face patterns by id $\in \{0, ..., 255\}$

unsuccessful for all but 1 traversal from S
(no 16×16×16-refinement)

For each traversal from S:

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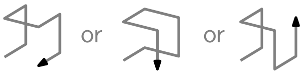
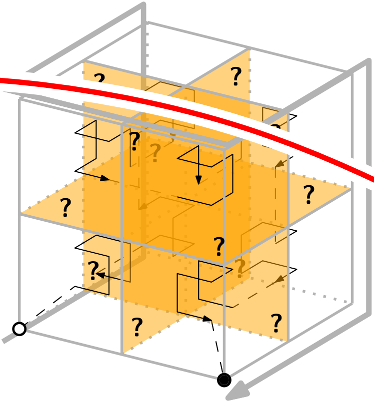


Finds ~~8384~~ ~~410~~ 1, for example:

$S' =$ ~~410~~ 48 ~~19 680~~

48 × ~~8384~~ = ~~402 432~~

traversals obtained by reflections and rotations



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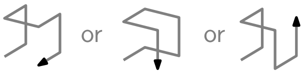

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$S' = \cancel{410}$ $\cancel{19\,680}$

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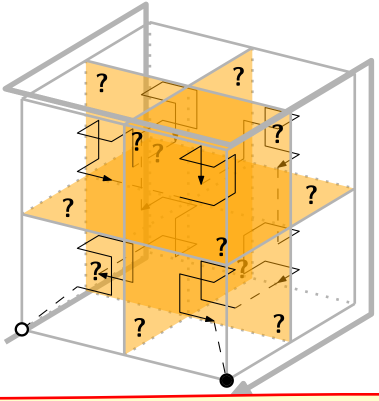

Identify face patterns by id $\in \{0, ..., 255\}$

no solution found →

no palindr., face-contin. octree traversal exists

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verify for each octant:

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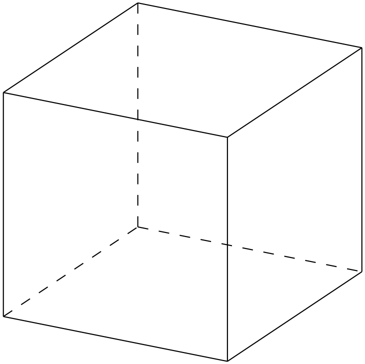
What about cubes?

3D Peano curve (applied by WEINZIERL 2009):



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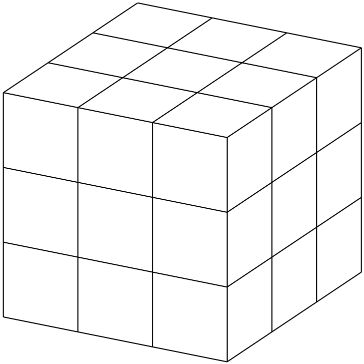

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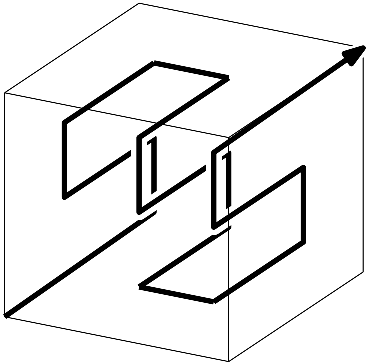

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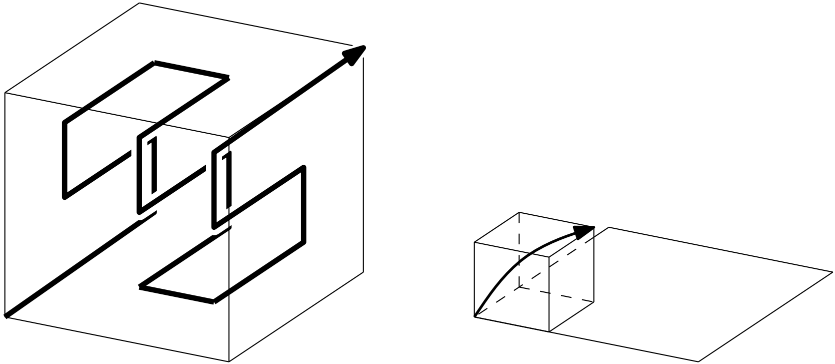

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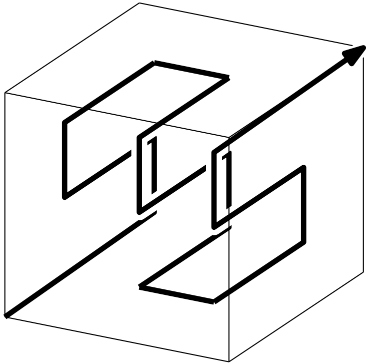

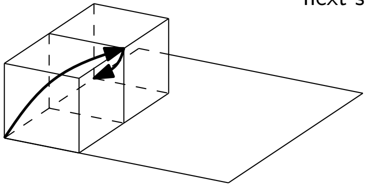

in each subcube, reflect pattern to connect to previous and next subcube

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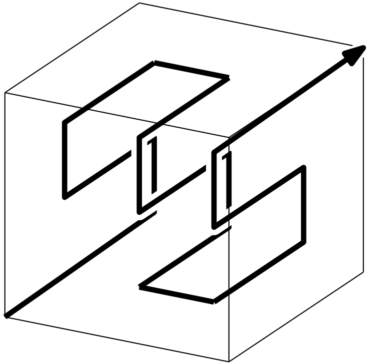

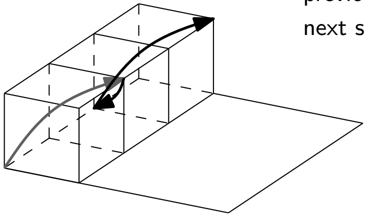

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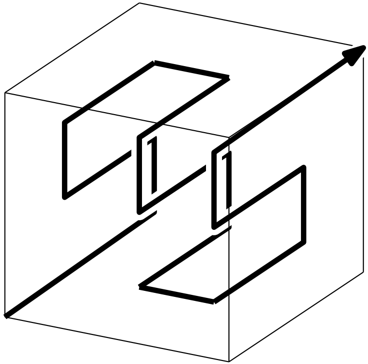

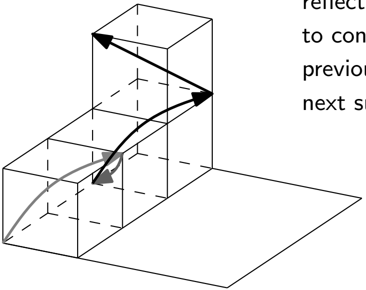

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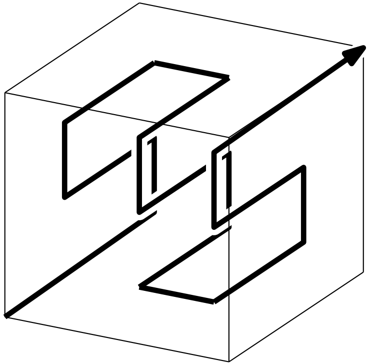

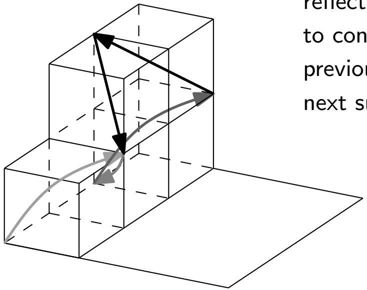

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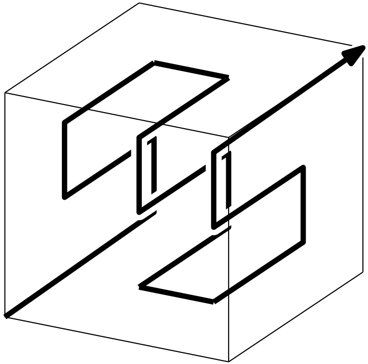

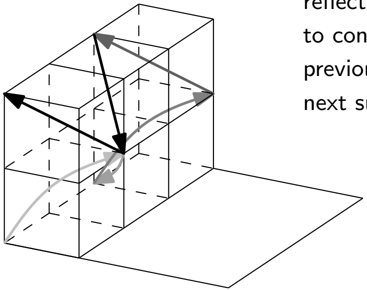

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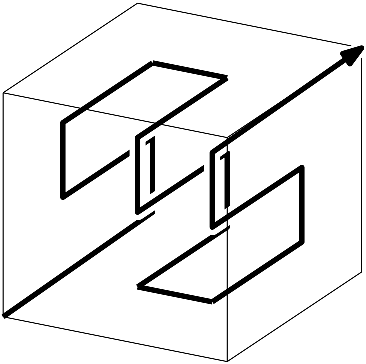

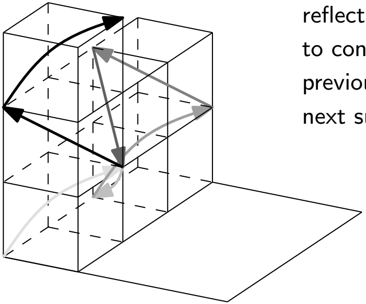

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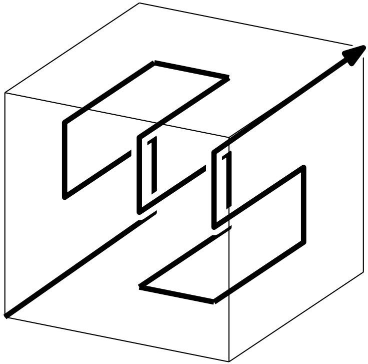

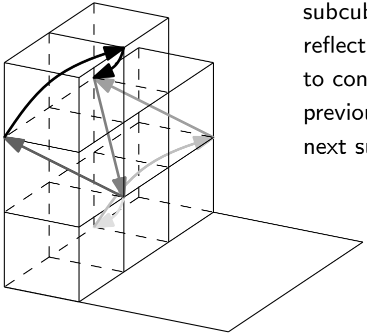

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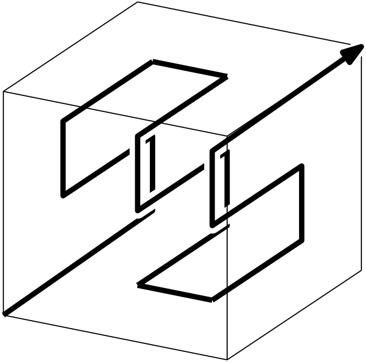

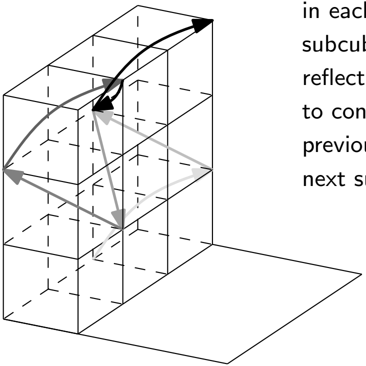

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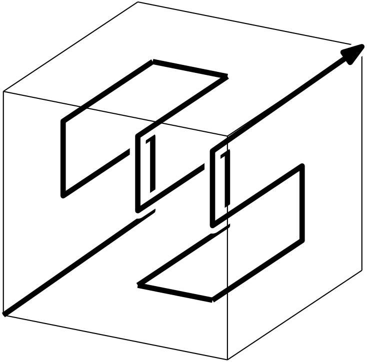

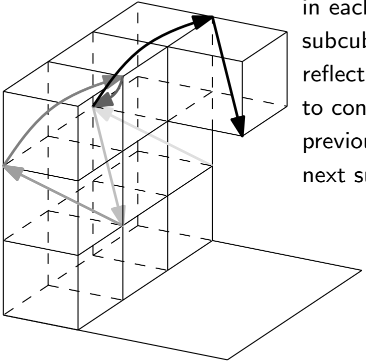

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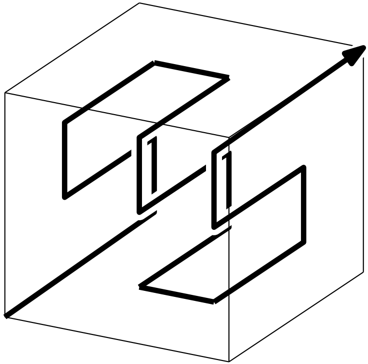

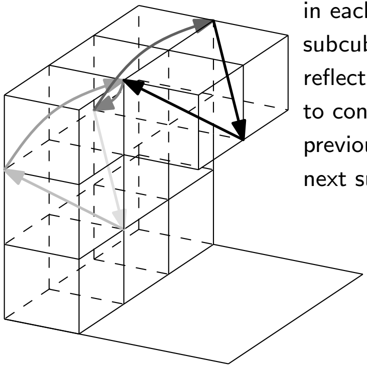

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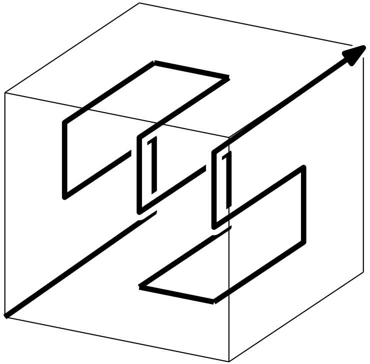
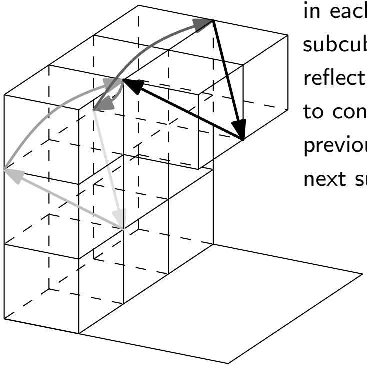




in each subcube, reflect pattern to connect to previous and next subcube

Desired: palindromic, face-continuous ~~oct~~ree traversal

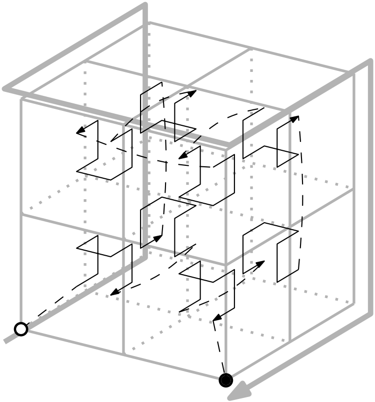
Cons:

- recursive subdivision into 27 subcubes → adaptive refinement less adaptive;
- partitions have larger surface-for-volume than with octree traversals (SASBURG 2011).

What about cubes?

Faloutsos's traversal (generalized from Faloutsos 1986)

Octants A and B share face f $\rightarrow$ traversal in B is reversed image of A under reflection in f



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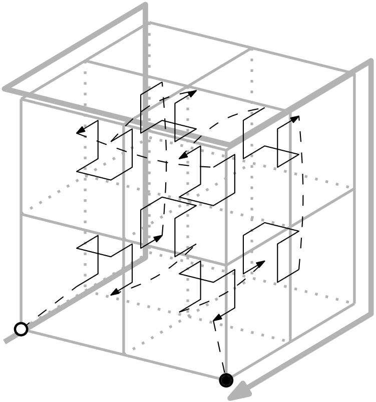
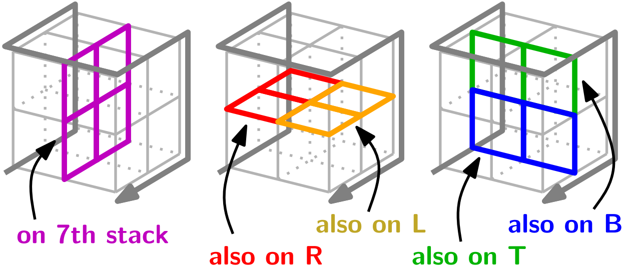


L: left
R: right
B: bottom
T: top
F: front
H: hind

stack for $\frac{\text{pushing}}{\text{popping}}$ vertices on (left / right / bottom / top / front / hind) face



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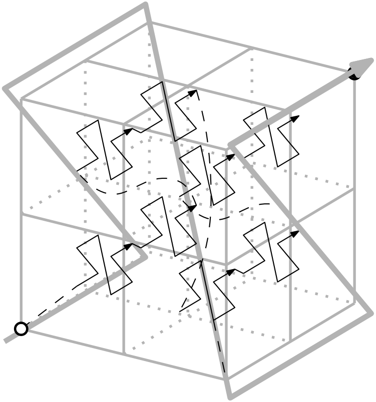
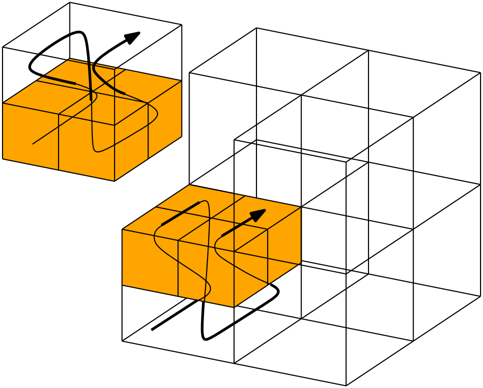
Morton order (Morton 1966): no rotations or reflections





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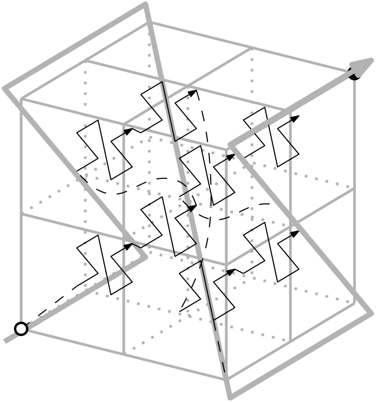

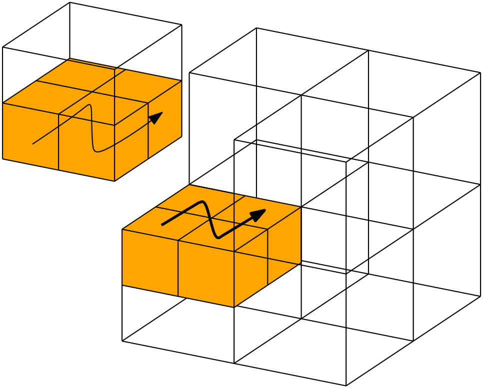

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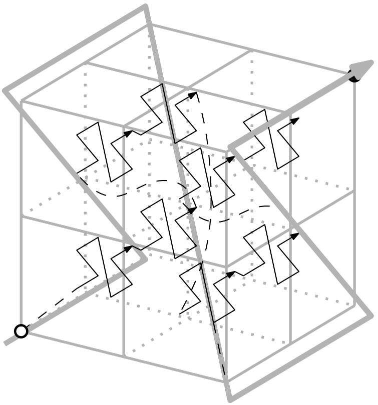

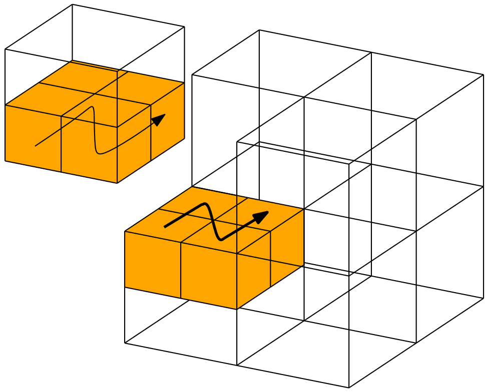

homodromic: 2nd traversal of face
= reverse of 1st *or the same*

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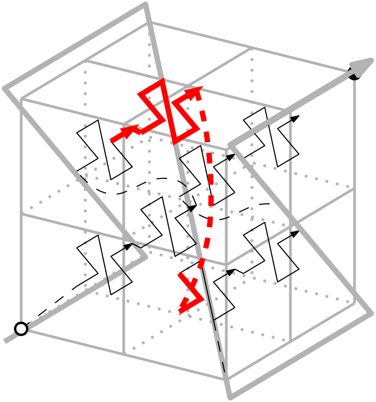

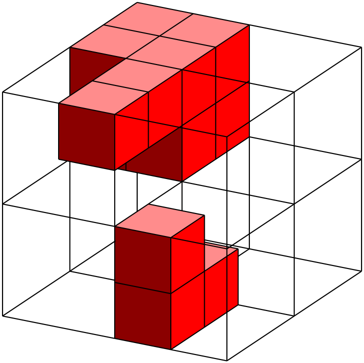

homodromic: 2nd traversal of face
= reverse of 1st *or the same*

quasi-face-continuous: interior of union of consecutive set of cells has $O(1)$ connected components.

Desired: ~~palindromic~~, ~~face-continuous~~ octree traversal

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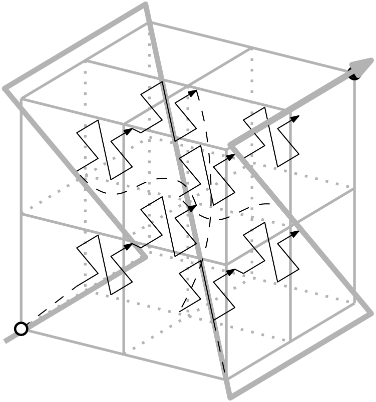

L:
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stack for $\frac{\text{pushing}}{\text{popping}}$ vertices on

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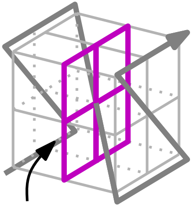
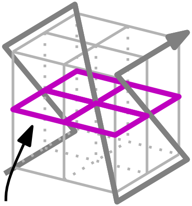
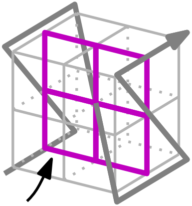


pushed onto R;
turned onto L
after 4th octant



pushed onto T;
turned onto B
aft. 2nd/6th oct.



pushed onto H;
turned onto F
aft. 1/3/5/7th

homodromic: 2nd traversal of face
= reverse of 1st *or the same*

quasi-face-continuous: interior of union of consecutive set of cells
has $O(1)$ connected components.

Desired: ~~palindromic~~, ~~face-continuous~~ octree traversal

What about cubes?

Desiderata:

- *octree traversal* (cubes/simplices recursively subdivided into 8 parts)
- *face-continuous*: consecutive cells share a face

  altern.: *quasi-face-cont.*: interior of union of set of consecutive cells has $O(1)$ components
- *palindromic*: for every set of adjacent cubes/simplices sharing a common face/edge, every subseq. traversal of common face/edge is reverse of previous traversal

  altern.: *homodromic*: subsequent traversals are reverse or same (more stack operations required)

octree trav.	non-face-continuous	quasi-face-continuous	face-continuous
non-homodromic			1000s of generalized Hilbert curves
homodromic		Morton traversal	
palindromic	Faloutsos's traversal		**computer says no**

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palindromic	Faloutsos's traversal	open How to recognize quasi-face-continuous traversals?	**computer says no**

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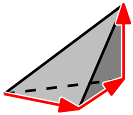

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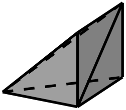

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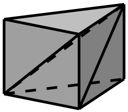

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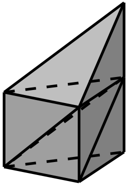

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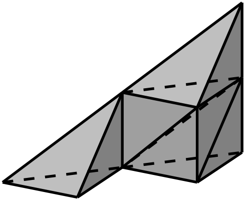

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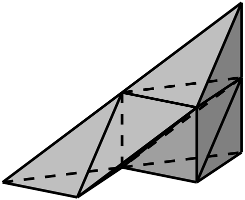

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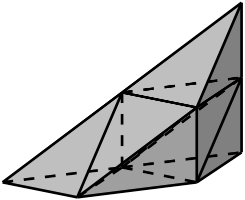

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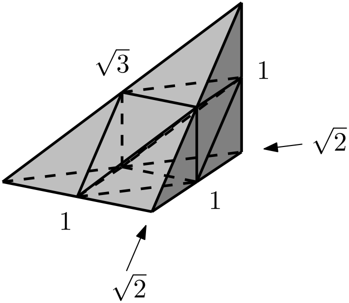

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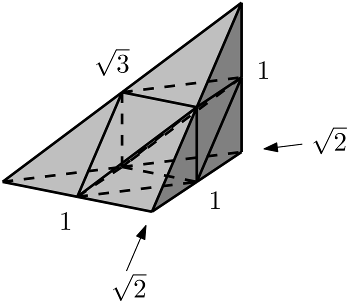

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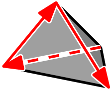

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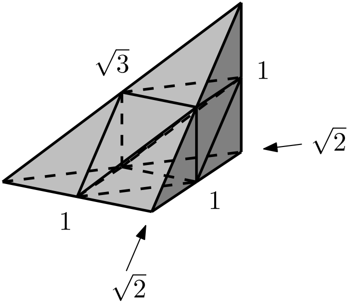

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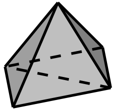

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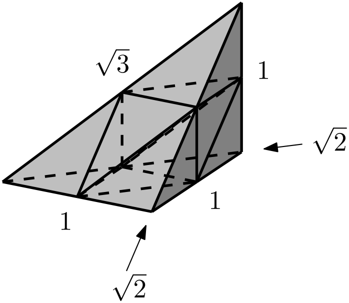

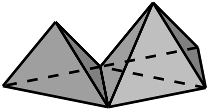

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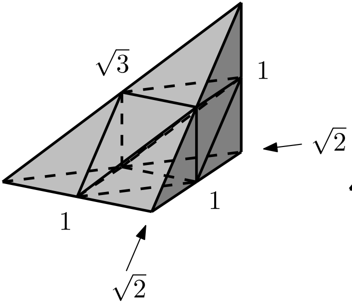

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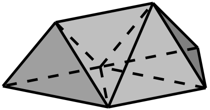

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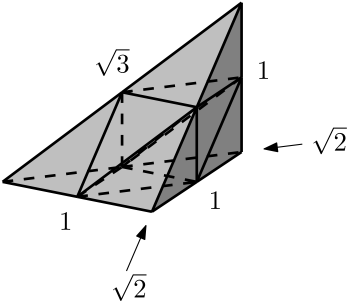

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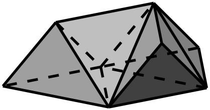

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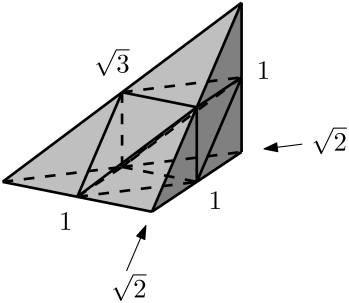

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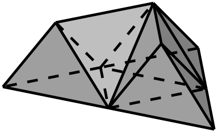

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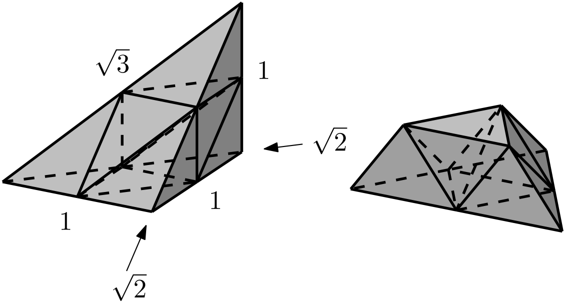

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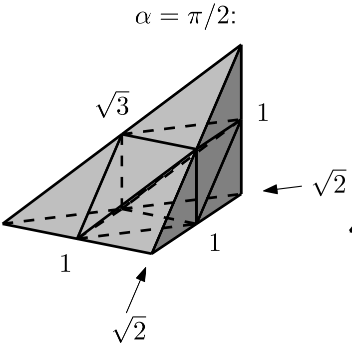

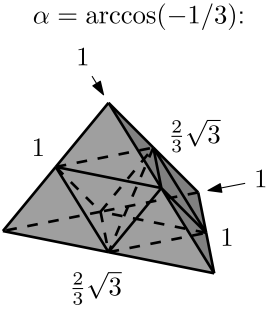

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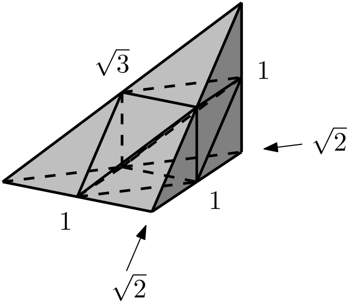

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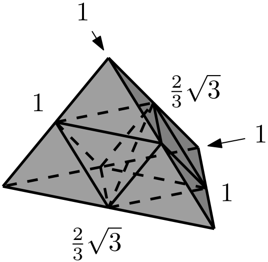

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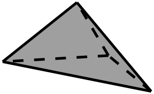

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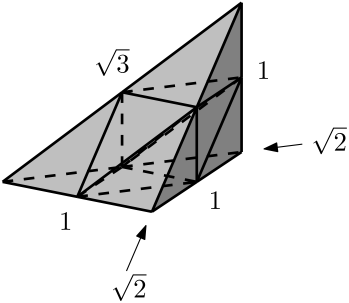

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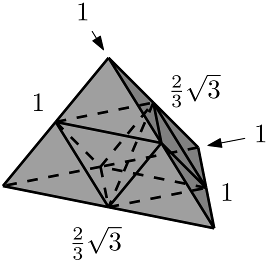

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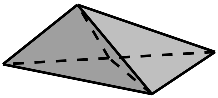

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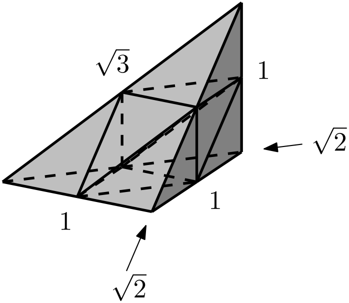

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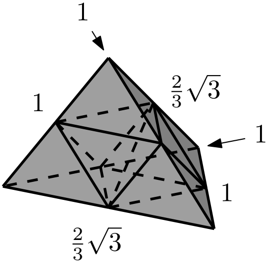

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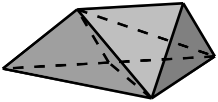

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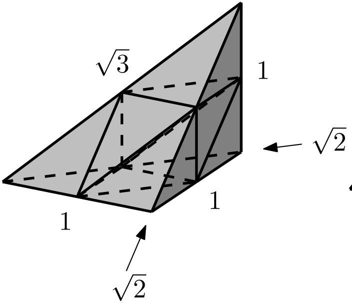

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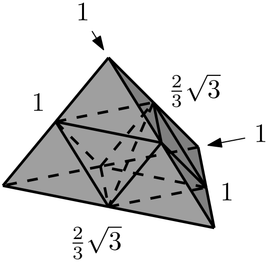

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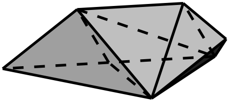

(special cases of Hill tetrahedra)

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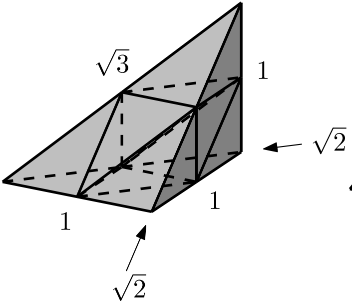

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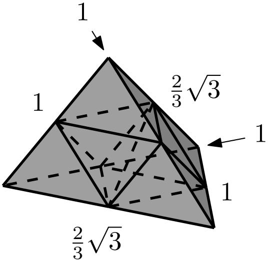

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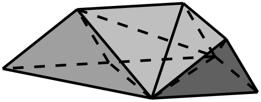

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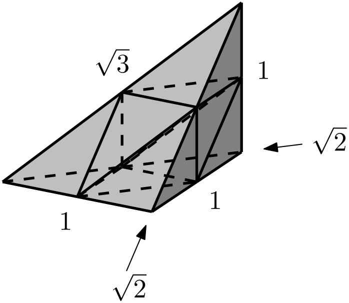

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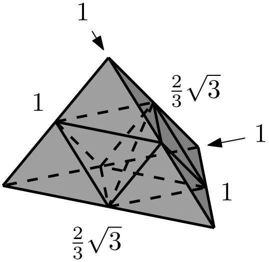

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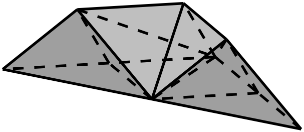

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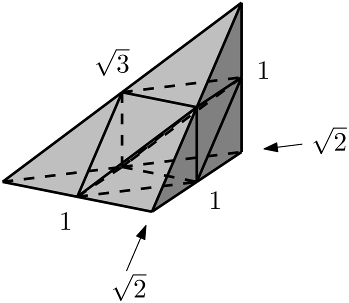

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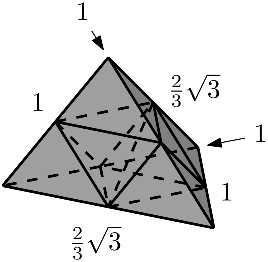

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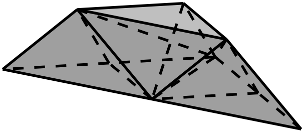

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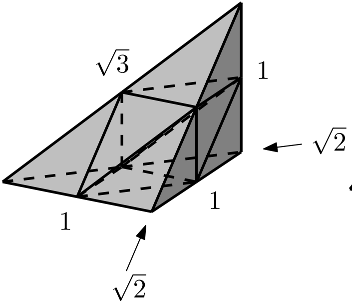

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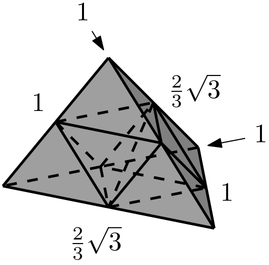

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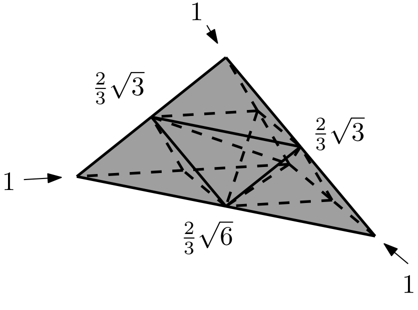

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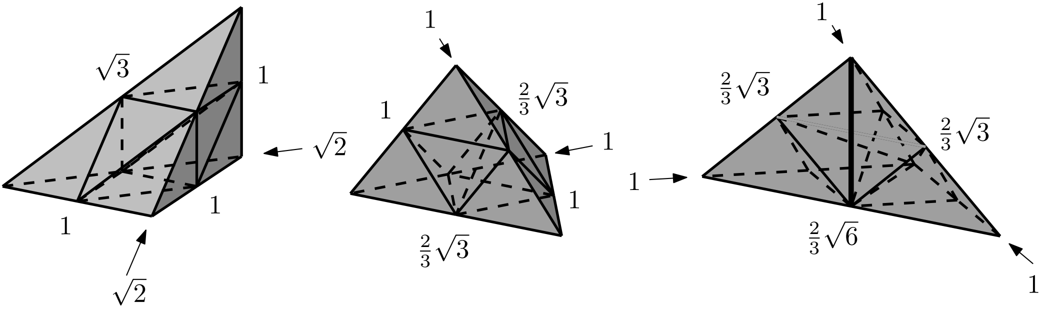

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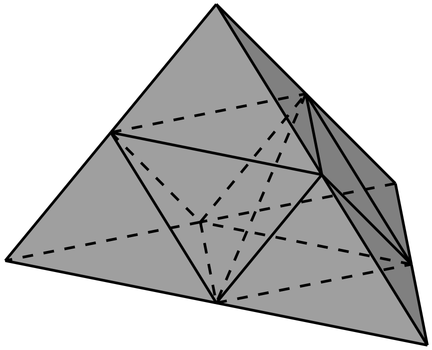
Only one way to subdivide into 8 equal, similar tetrahedra.



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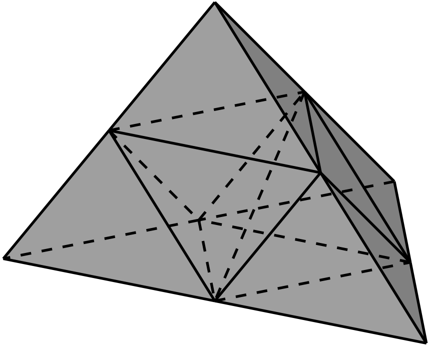
Four subtetrahedra share a face with only one other $\rightarrow$ no face-continuous traversal



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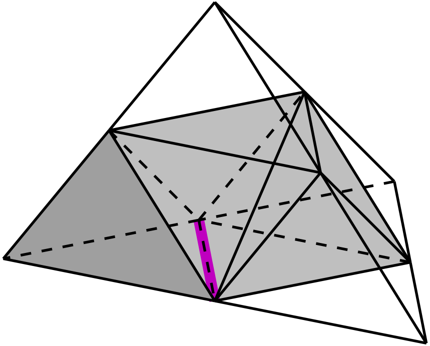

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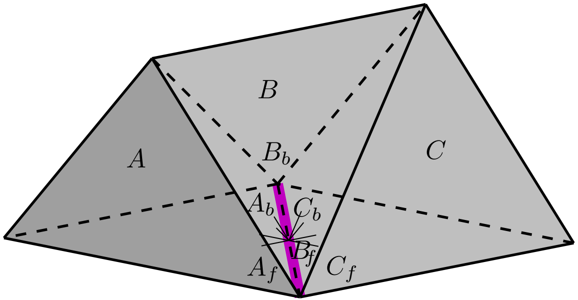

X_f, X_b: subtetrahedron of X in the front, back

Palindromic implies: $(A_f \prec A_b) = (B_b \prec B_f) = (C_f \prec C_b)$

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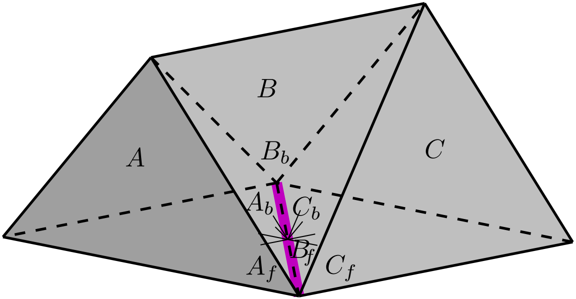

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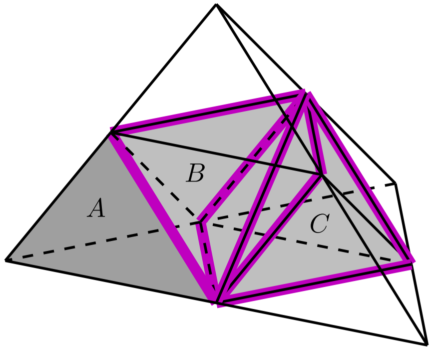


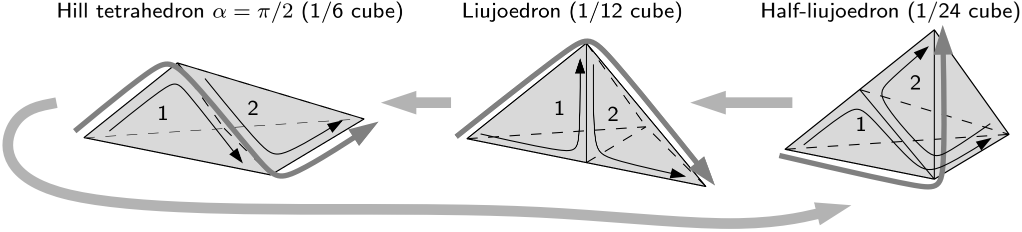
$\rightarrow$ between traversals of A and C, stack with vertices of common edge must be reversed

$\rightarrow$ B must be traversed between A and C

$\rightarrow$ combine conditions on all edges $\rightarrow$ $\lightning$ no palindromic traversal possible

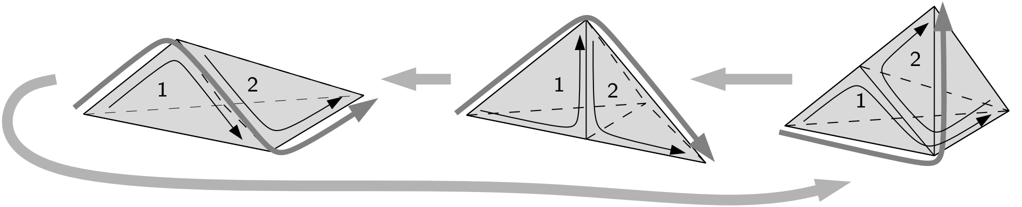
(still hope for: quasi-face-continuous, homodromic traversal)

Tetrahedral meshes: liujoedron bisection scheme



quasi-face-continuous, homodromic

Hill tetrahedron $\alpha = \pi/2$ (1/6 cube) Liujoedron (1/12 cube) Half-liujoedron (1/24 cube)



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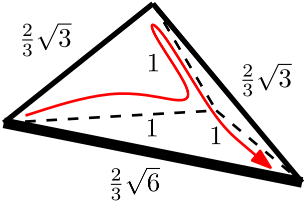

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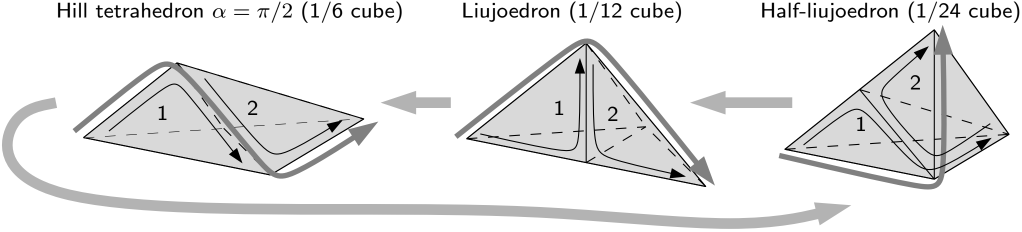

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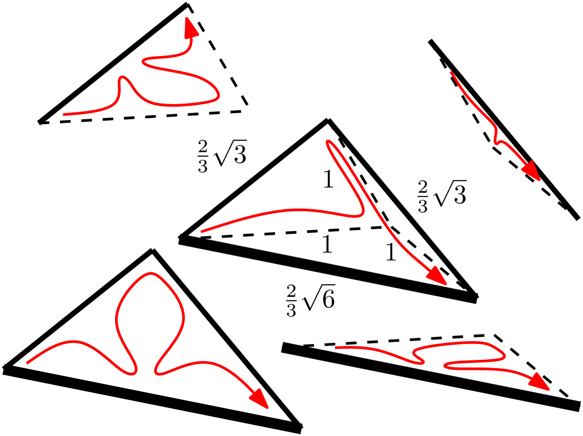

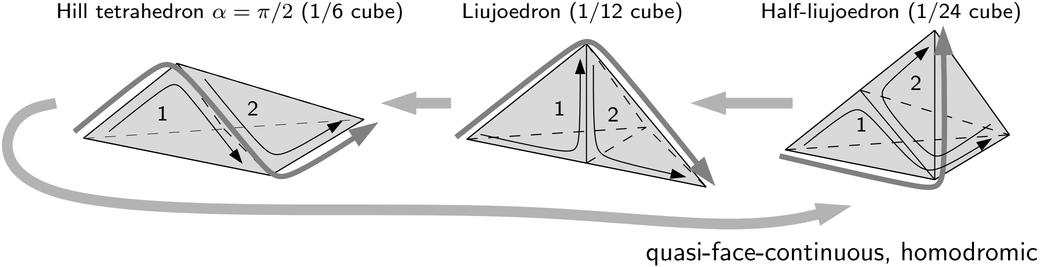
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1
2
1
2
1
2
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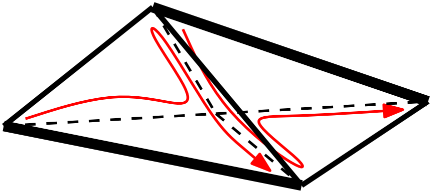

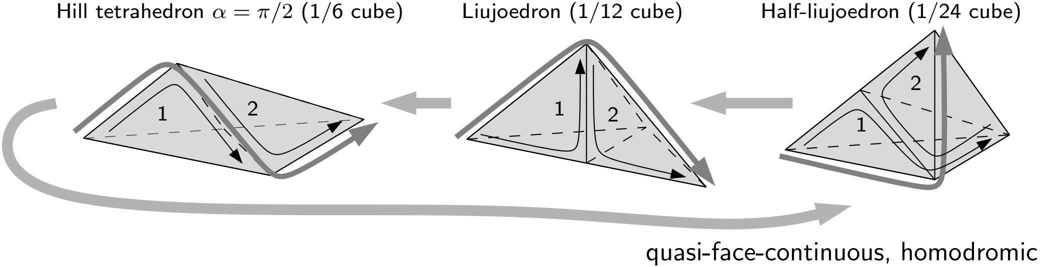
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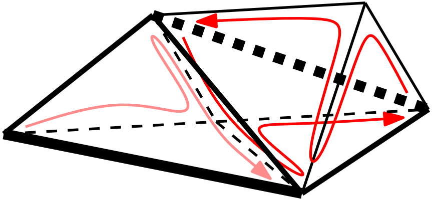

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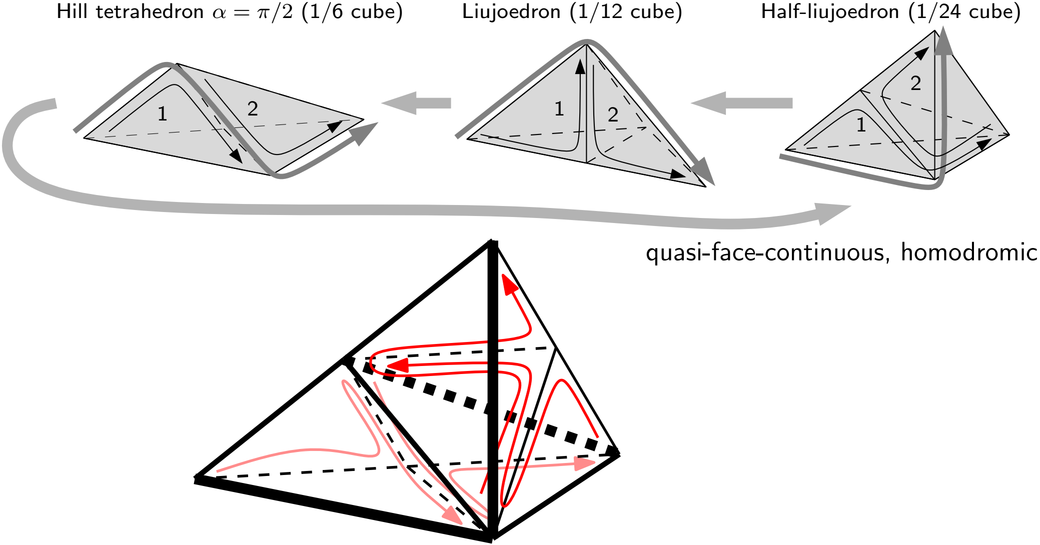

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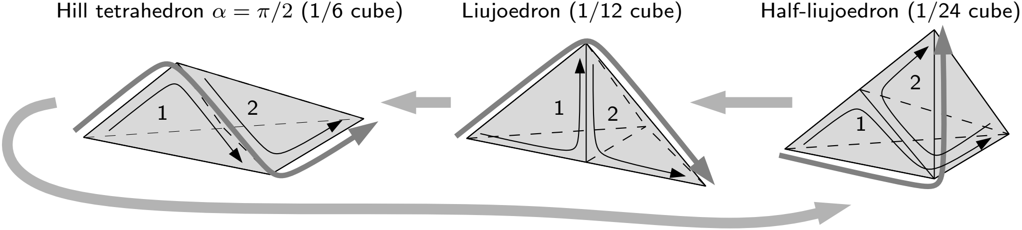

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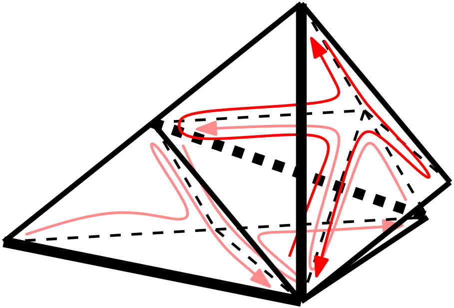

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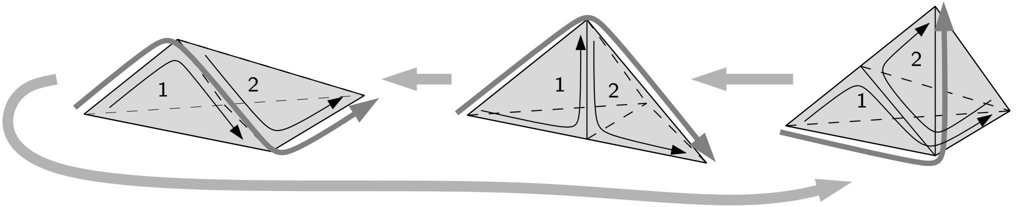

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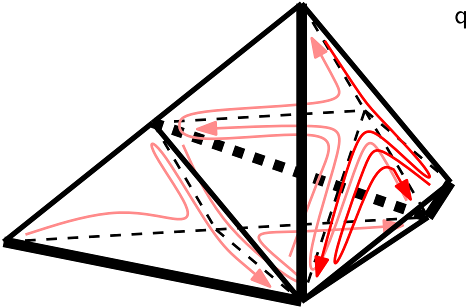

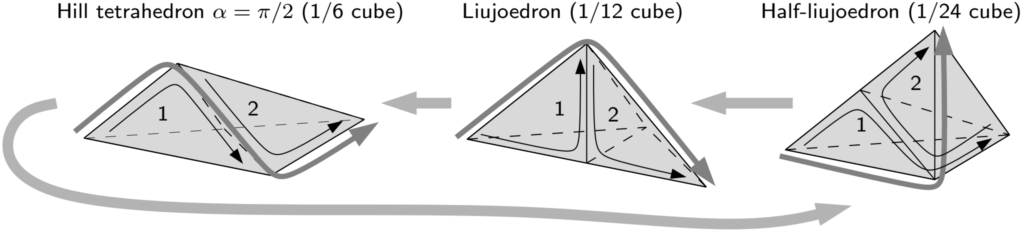

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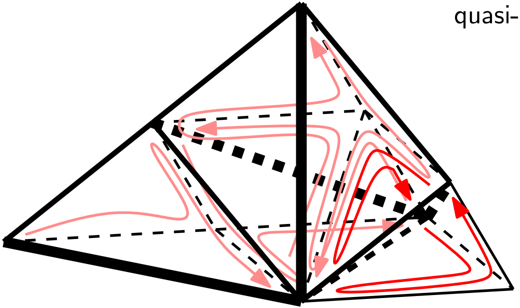

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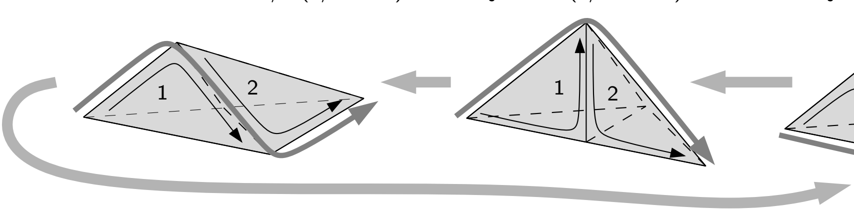

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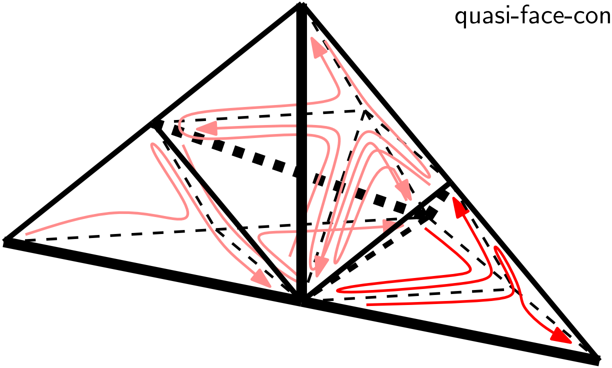

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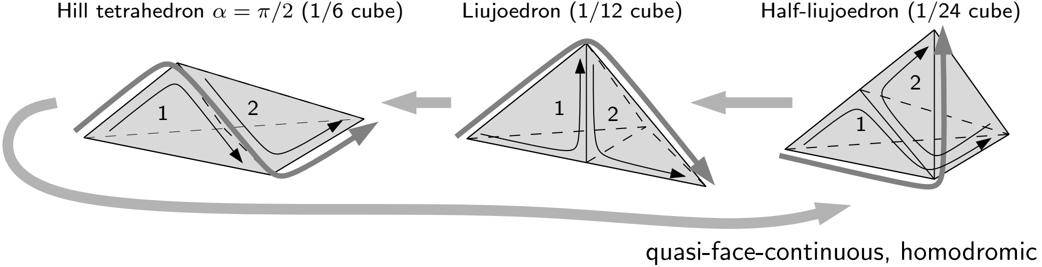

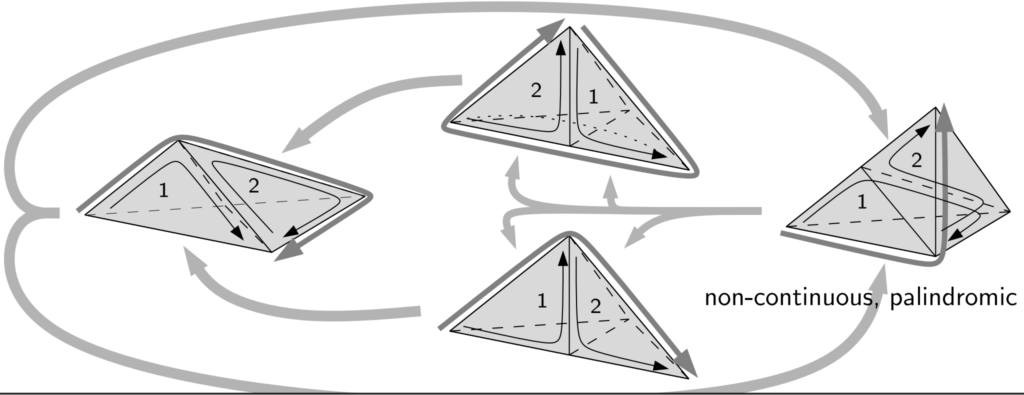

Summary: results on octree traversals

+	traversal known
− (light red)	impossible (almost proven)
− (red)	impossible

Cubes

	...-continuous non-	quasi-	face-
non-homodr.	+	+	+
homodromic	+	+	−
palindromic	+	?	−

Hill-tetrahedra $(\alpha \neq \pi/2)$

	...-continuous non-	quasi-	face-
n.-h.	+	?	−
h.dr.	?	?	−
p.dr.	−	−	−

Bisected tetrahedra

	...-continuous non-	quasi-	face-
n.-h.	+	+	−
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Open problems:

- insightful proof of negative results on cubes
- meaningful surface-to-volume measures—and how to compute them?
- what tetrahedra are reptiles?
- (traversals for 1/6, 1/12, 1/24 cube tetrahedra that do not follow bisection scheme?)
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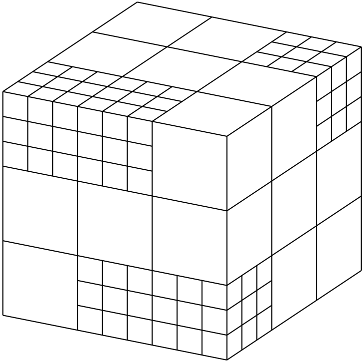
THANK YOU FOR YOUR ATTENTION

Bonus slide: 3D Peano traversal



Solid object with heat sources and sinks on boundary;
simulate heat distribution on vertices, flow through faces by repeatedly iterating over all cells.

Requires:

- storing vertex/edge/face values between iterations
- repeated access to v/e/faces on boundaries between cells
- adaptive refinement of the mesh

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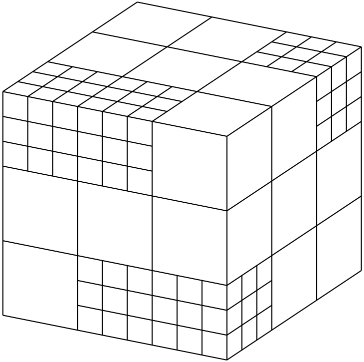

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WEINZIERL 2009:

current vertex/edge/face values popped from *input stack* when first visited;
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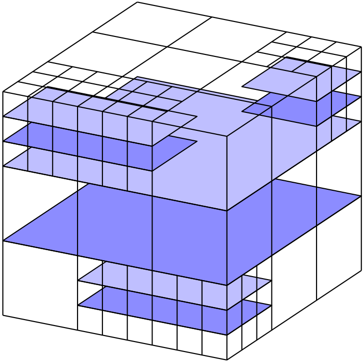
between first and last visits to a vertex/edge/face, value stored on intermediate stacks:

Bonus slide: 3D Peano traversal



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- two stacks for horizontal faces (one for odd heights, one for even heights)

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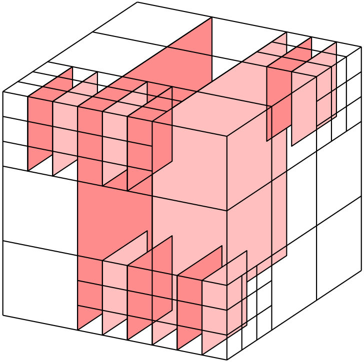

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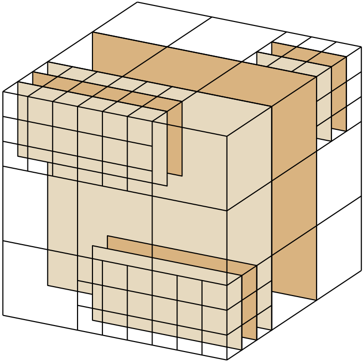
Bonus slide: 3D Peano traversal



Solid object with heat sources and sinks on boundary; simulate heat distribution on vertices, flow through faces by repeatedly iterating over all cells.

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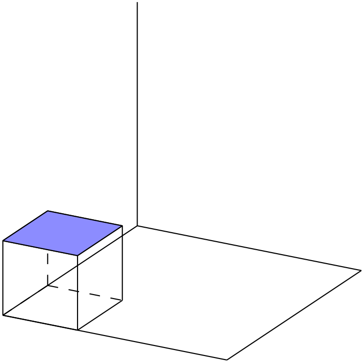

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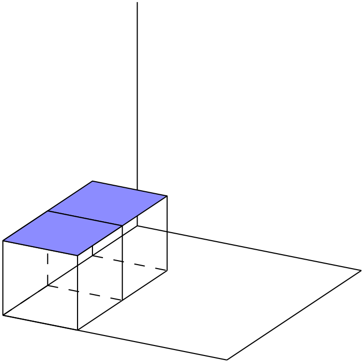

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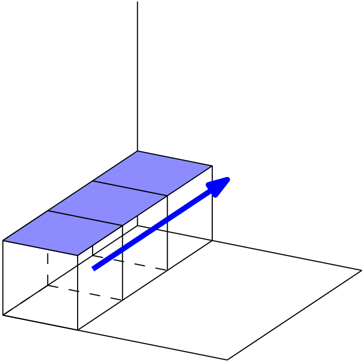

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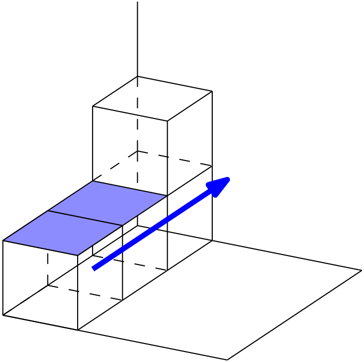

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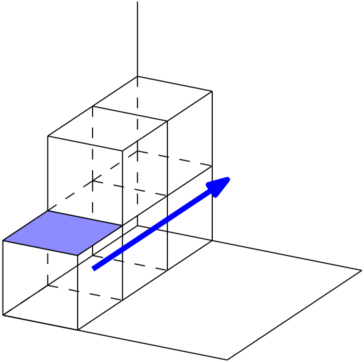

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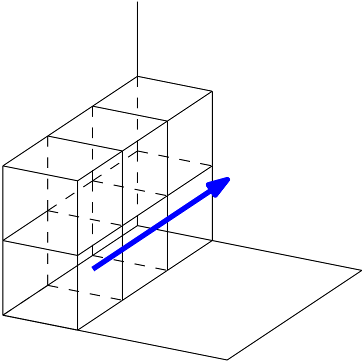

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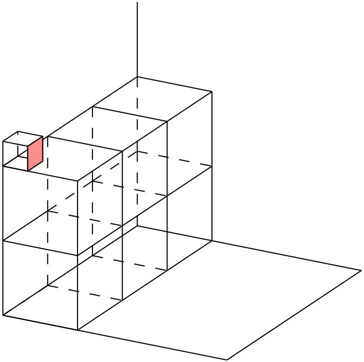

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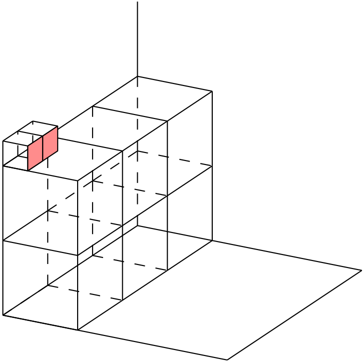

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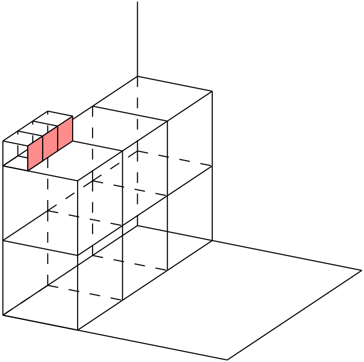

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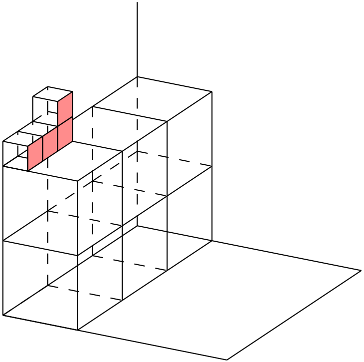

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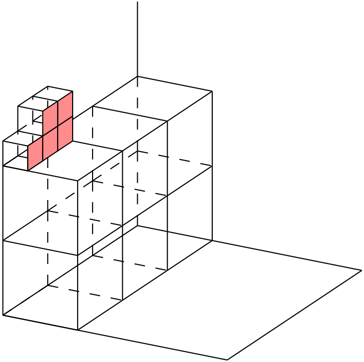

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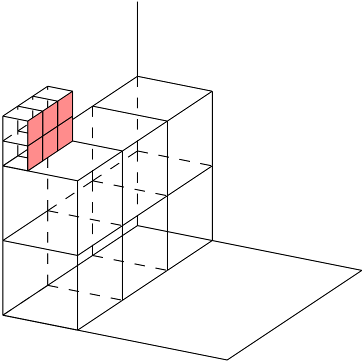

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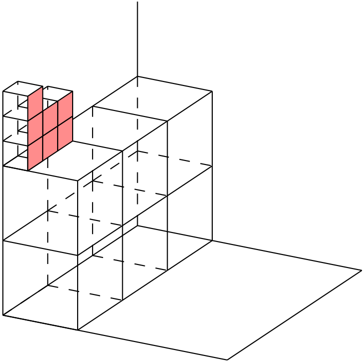

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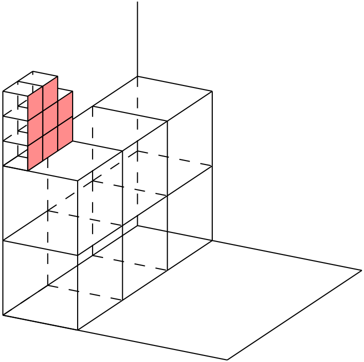

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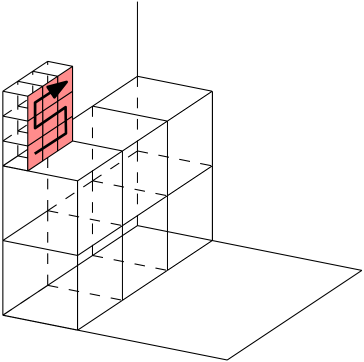

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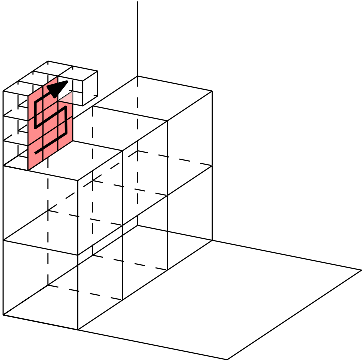

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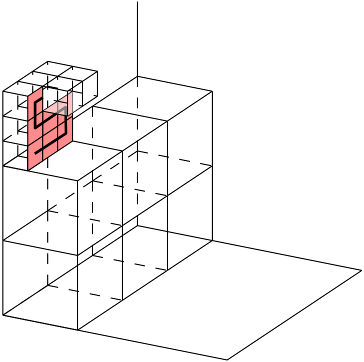

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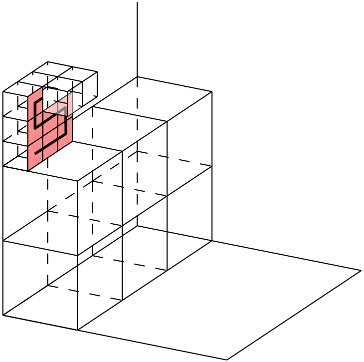

- cache-efficient!
- adaptive refinement = some pushes on stacks:
  no complicated vertex/edge/face index!
- easy to parallelize! each processor gets part of traversal
  (only diff: values for variables on boundary with other processor are read/written to different stacks)

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