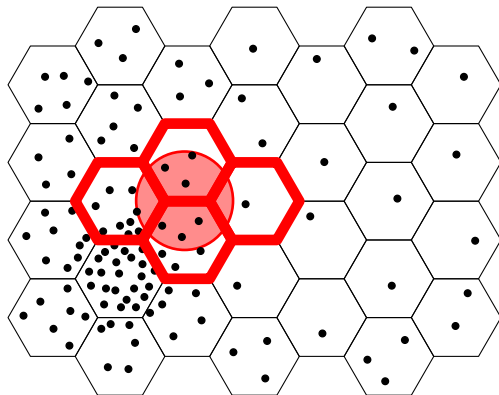


Circular range query: report all points inside query circle Q

non-recursive tiling

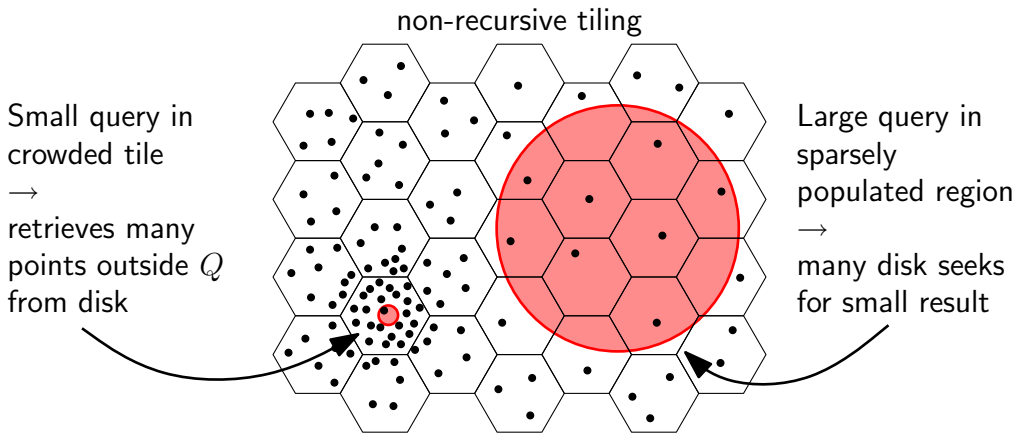


Data structure: store points tile by tile; each tile = contiguous block on disk

Circular range query: report all points inside query circle Q

Query algorithm: retrieve every tile intersecting Q (one disk seek per tile)

Recursive tilings and space-filling curves with little fragmentation



Data structure: store points tile by tile; each tile = contiguous block on disk

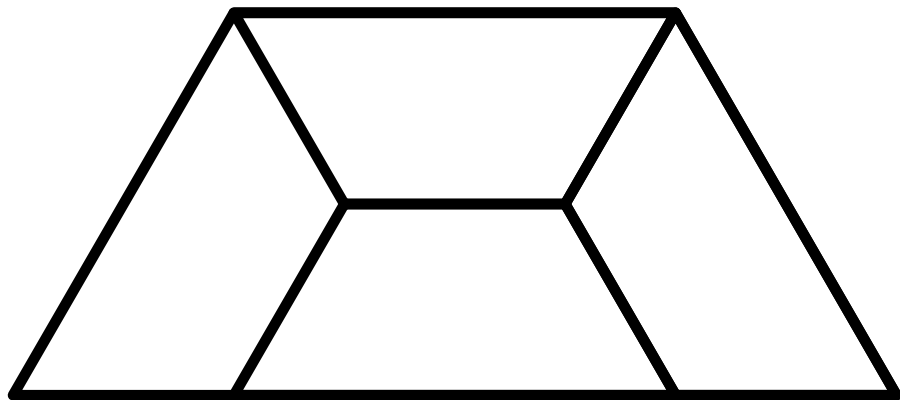
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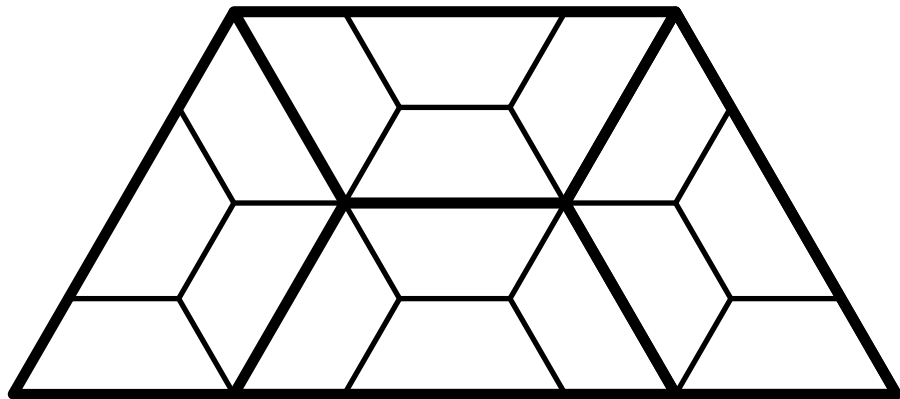
recursive tiling



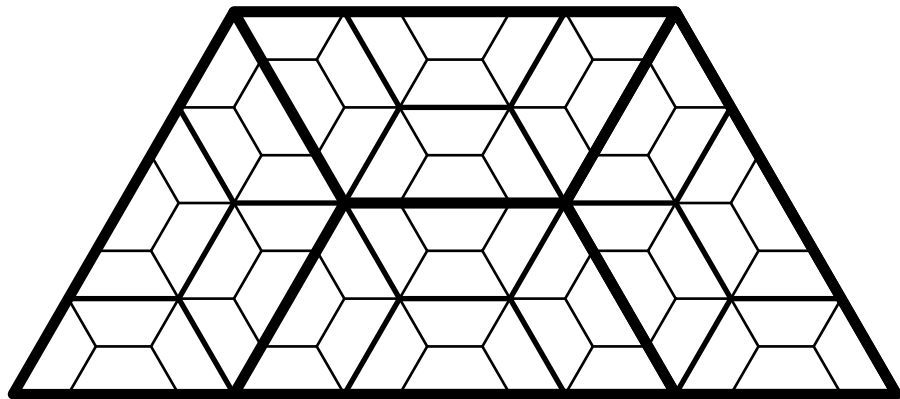
recursive tiling



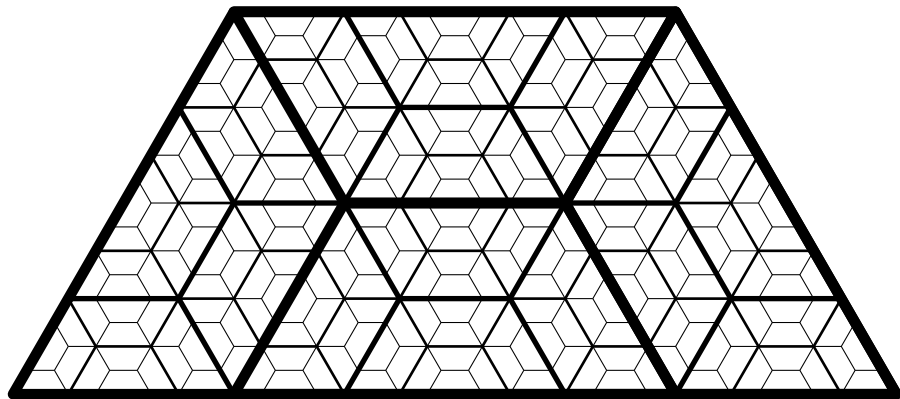
recursive tiling



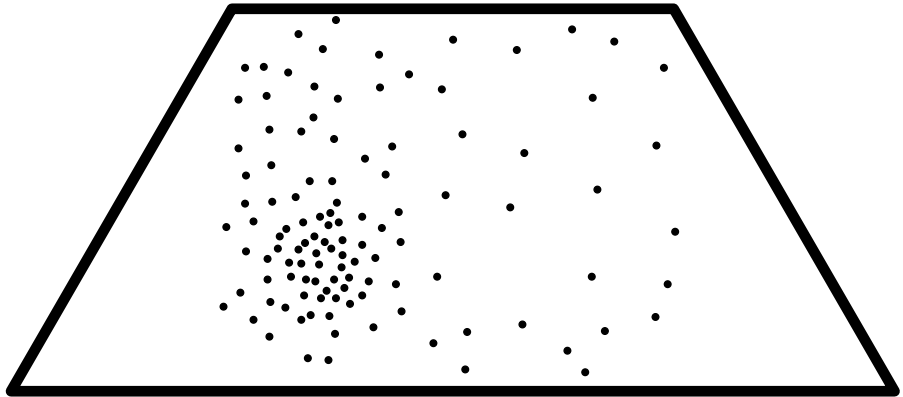
recursive tiling



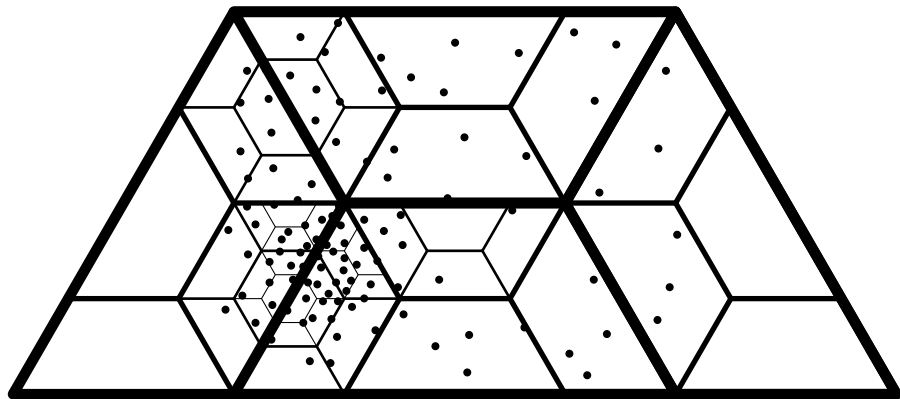
recursive tiling



recursive tiling

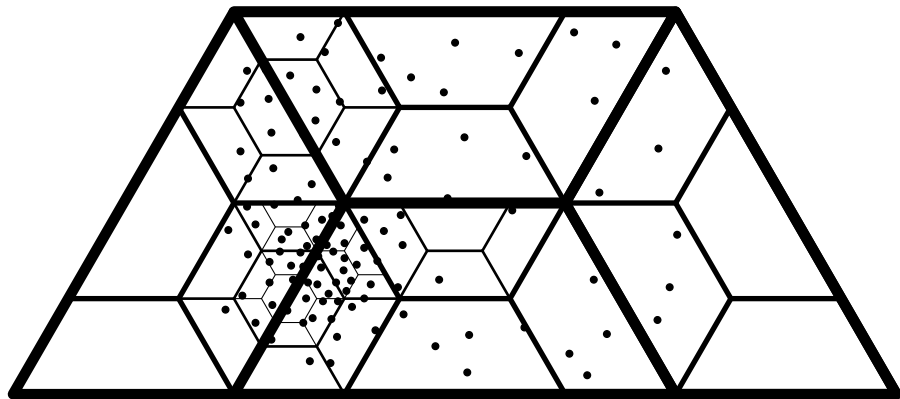


recursive tiling



Data structure: store points tile by tile; each tile = contiguous block on disk
(similar to linear quadtree)

recursive tiling



Data structure: store points tile by tile; each tile = contiguous block on disk

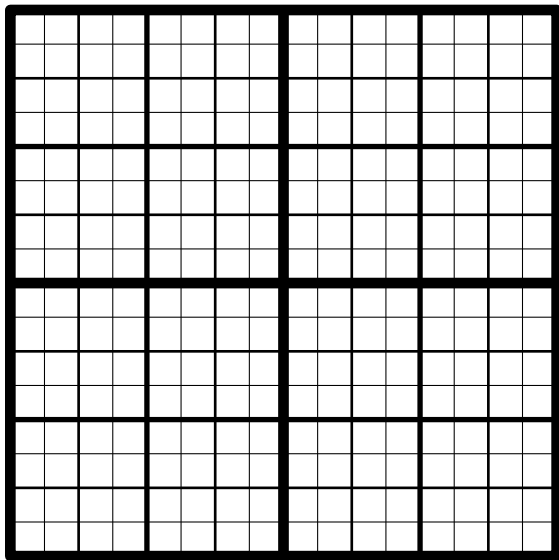
Goal 1: a tiles suffice to cover any disk-shaped query range Q (seek time)

Goal 2: the tiles that cover Q have total area at most $c \cdot \text{area}(Q)$ (read time)

Arrwid number =

smallest a such that there is a constant c such that $\forall Q$ both goals achieved

The Arrwid number of a tiling with squares

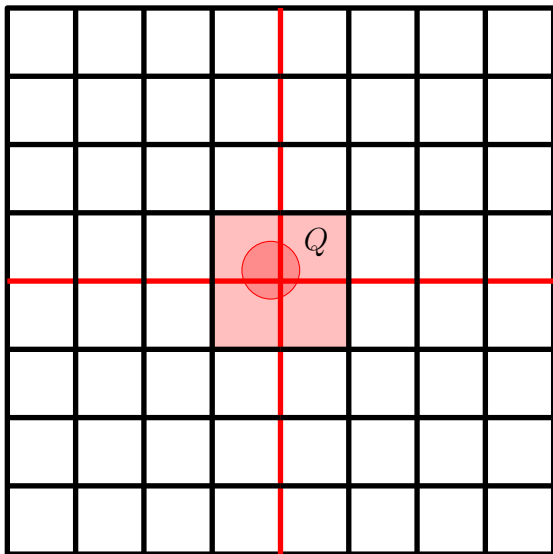


Arrwid nr. of square tiling ≤ 4 :

Arrwid nr. of square tiling > 3 :

Arrwid number = smallest a such that \exists constant c such that any disk Q is covered by $\leq a$ tiles of total area $\leq c \cdot \text{area}(Q)$

The Arrwid number of a tiling with squares



Arrwid nr. of square tiling ≤ 4 :

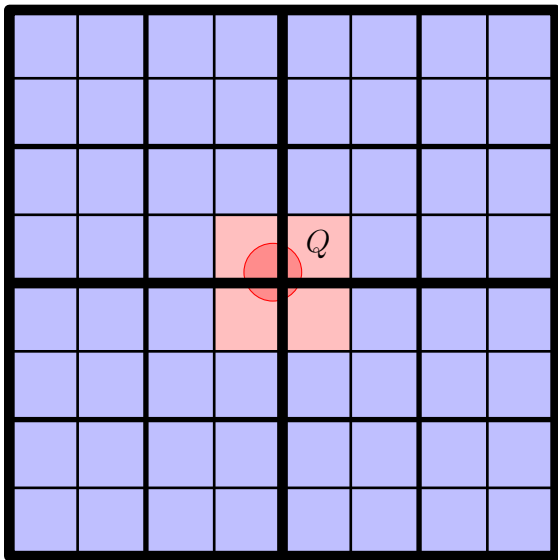
For disk with radius r , consider grid with tile width $\geq 2r, < 4r$:

disk intersected by ≤ 2 grid lines;
cover by ≤ 4 tiles meeting there.

Arrwid nr. of square tiling > 3 :

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The Arrwid number of a tiling with squares



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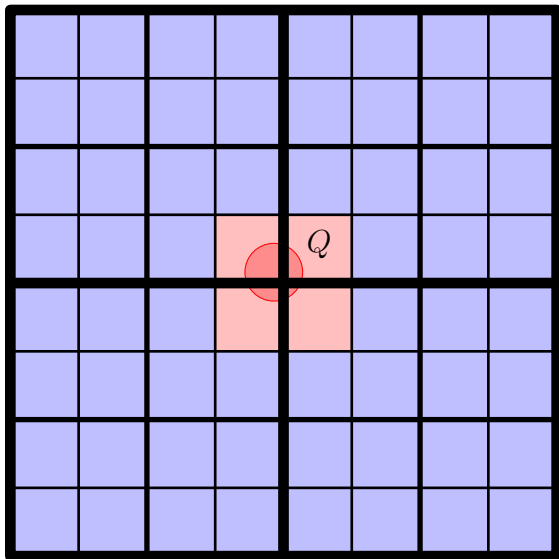
disk intersected by ≤ 2 grid lines;
cover by ≤ 4 tiles meeting there.

Arrwid nr. of square tiling > 3 :

To cover with ≤ 3 tiles,
need **common ancestor** of red tiles,
can be much bigger than Q
 \rightarrow no constant c .

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The Arrwid number of a tiling with squares



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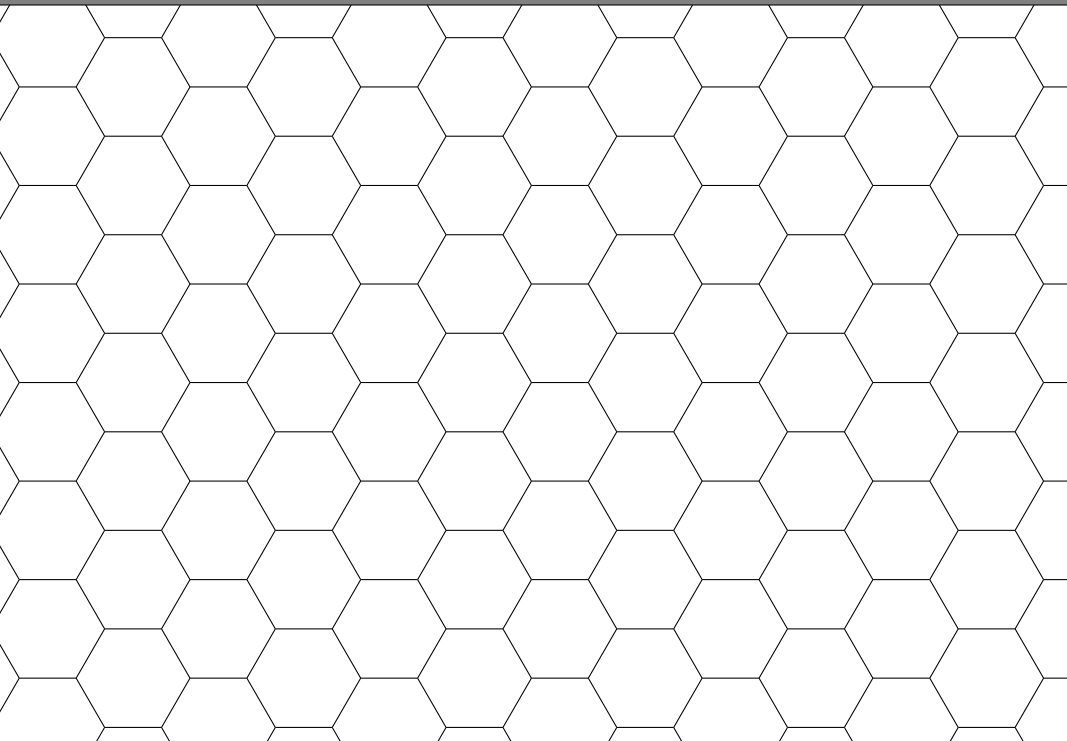
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Arrwid nr. \approx degree of vertices

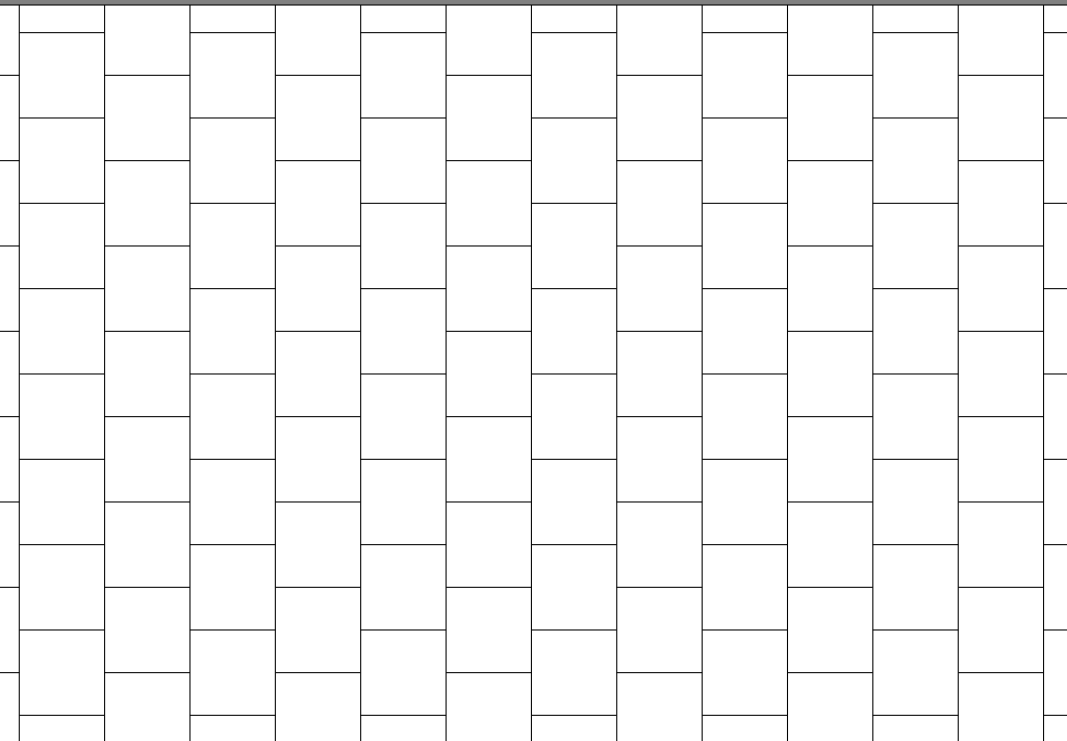
Are there recursive tilings with vertex degree 3?

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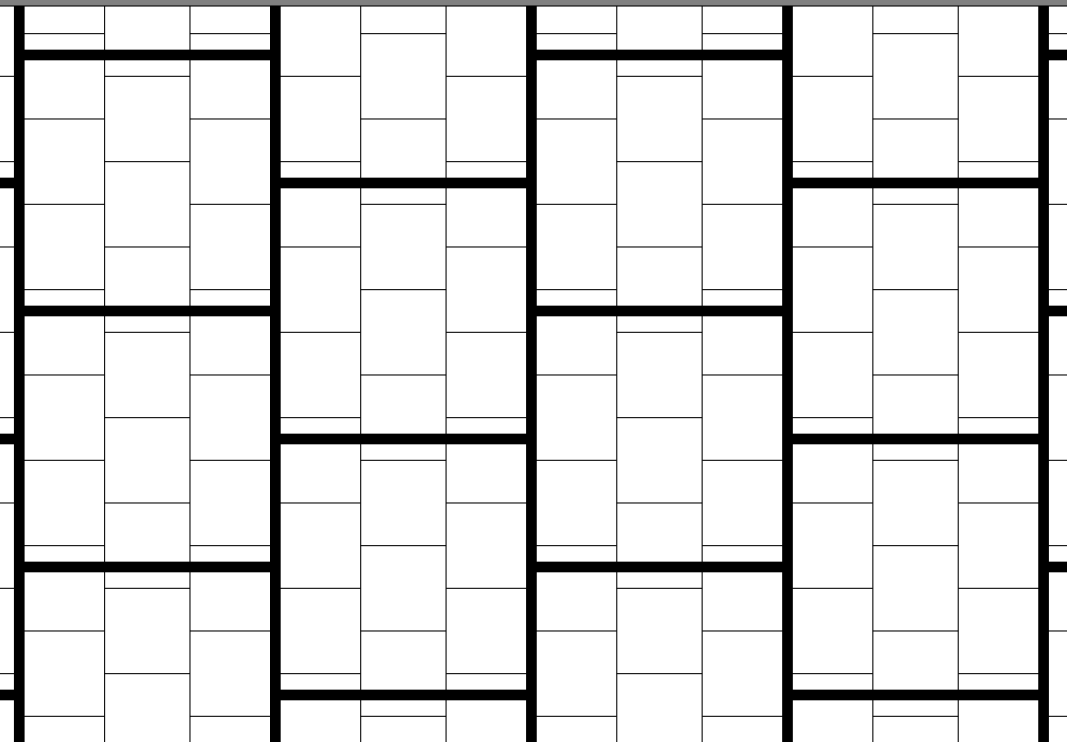
From non-recursive tilings...



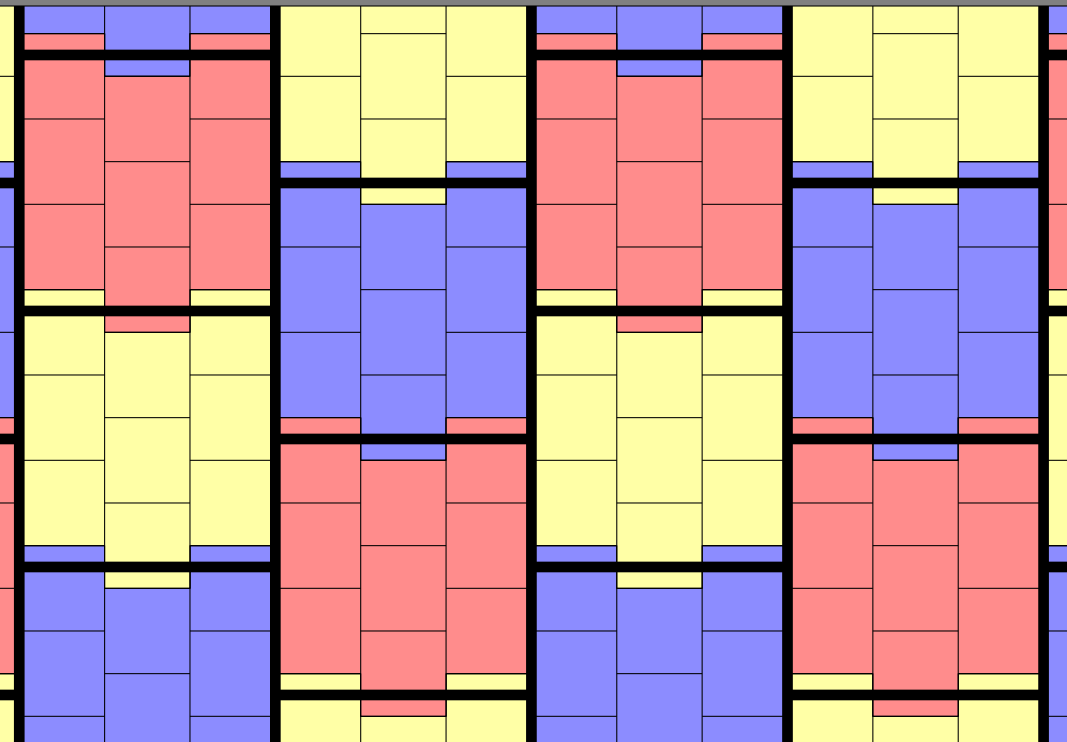
From non-recursive tilings...



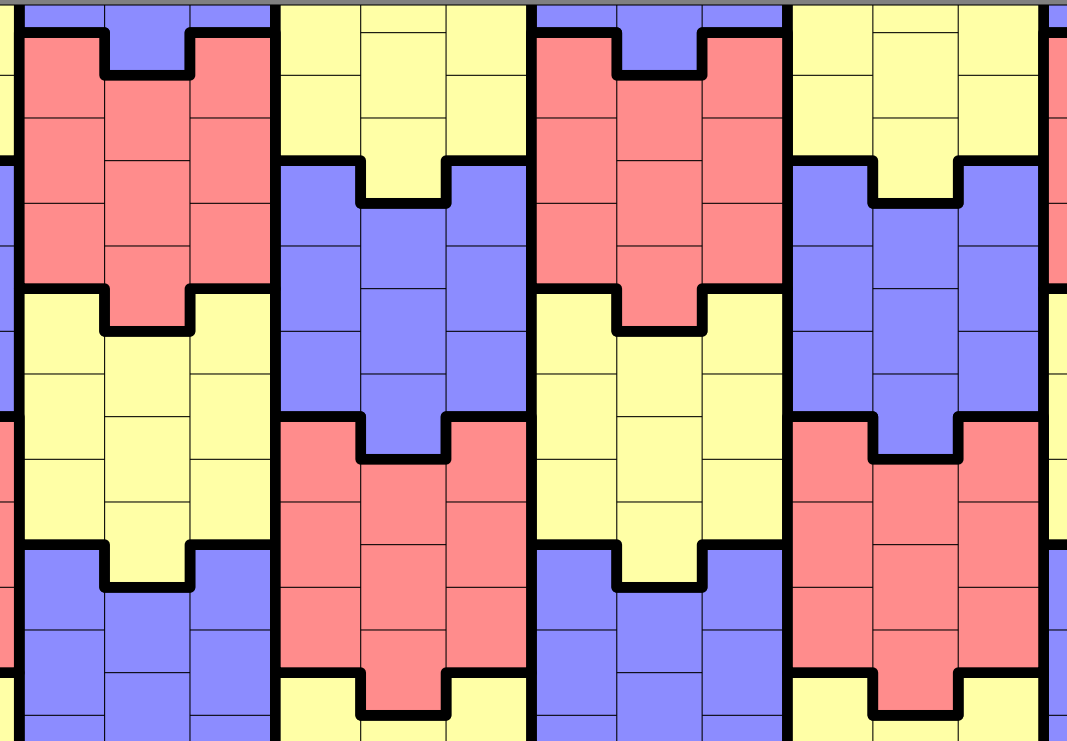
From non-recursive tilings to recursive tilings



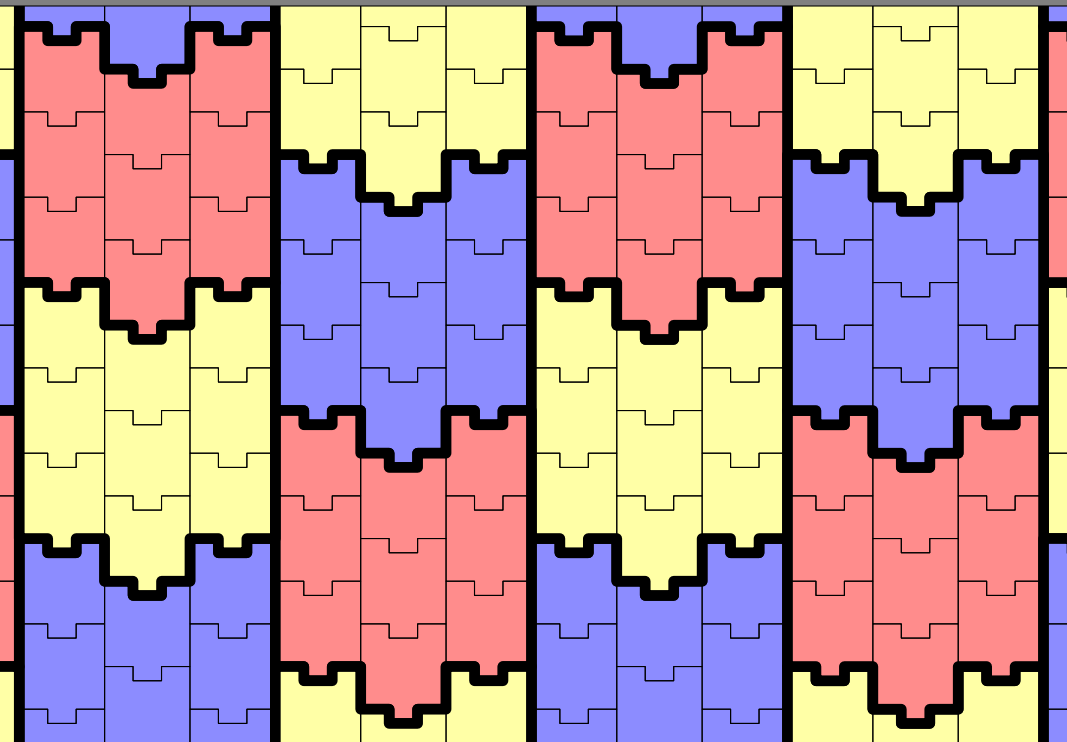
From non-recursive tilings to recursive tilings



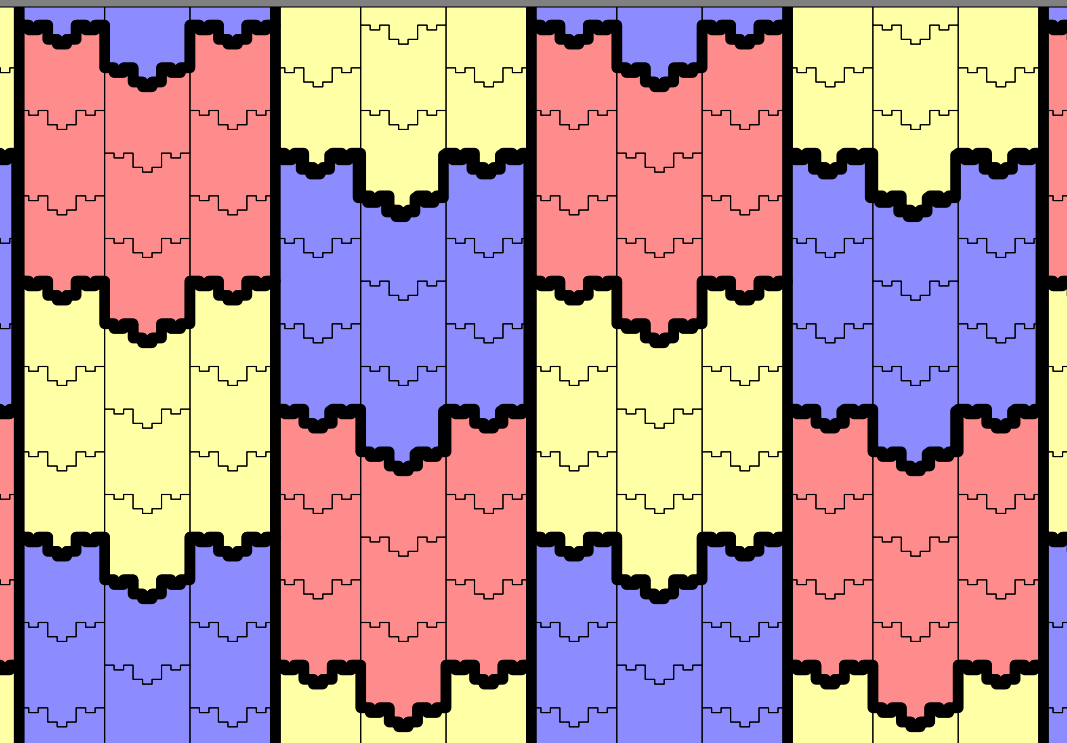
From non-recursive tilings to recursive tilings



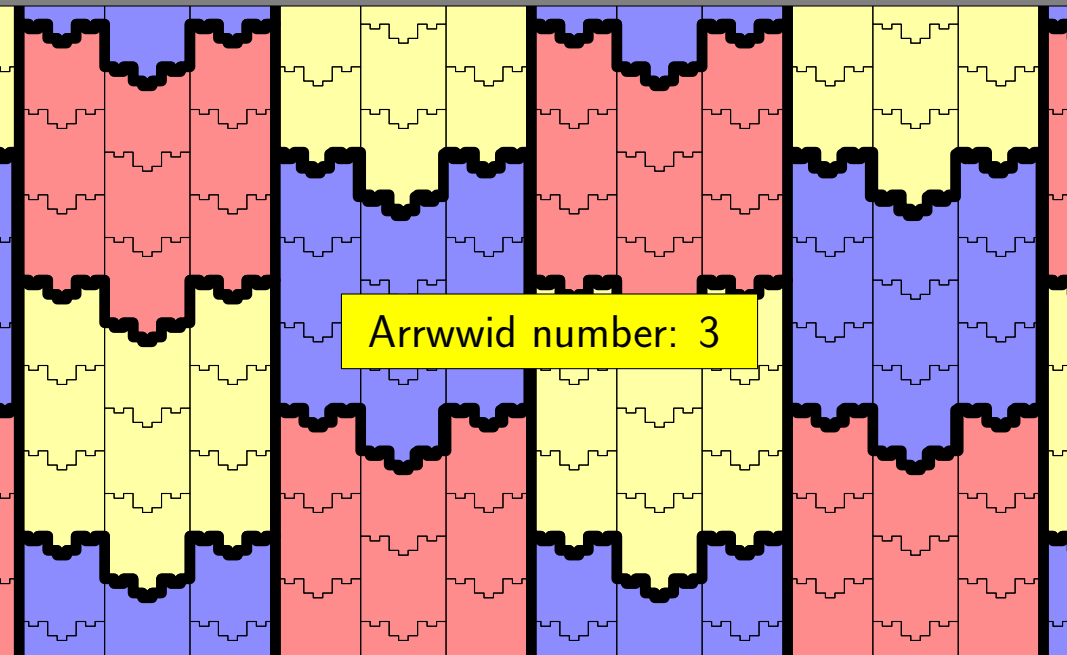
From non-recursive tilings to recursive tilings



From non-recursive tilings to recursive tilings



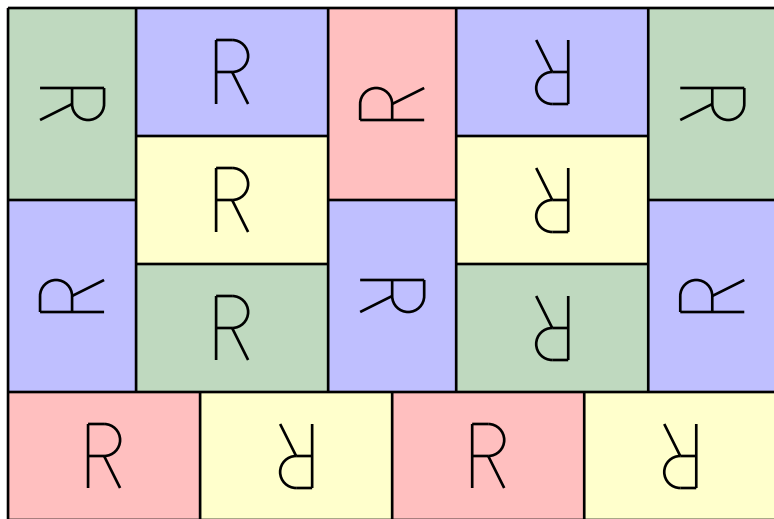
From non-recursive tilings to recursive tilings



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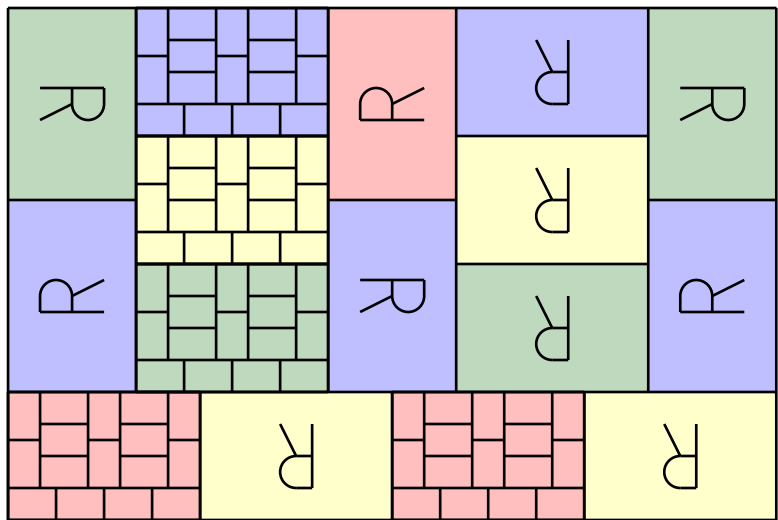
Daun tiling: Arrwwid nr. 3 and "easy" tiles

R:



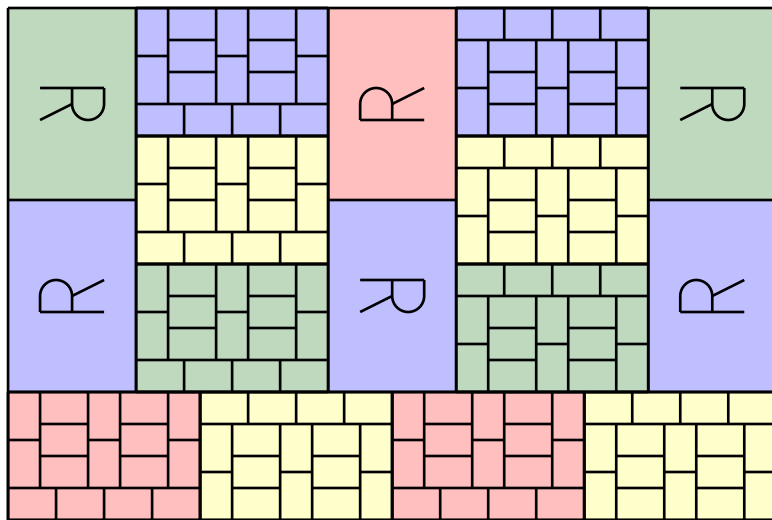
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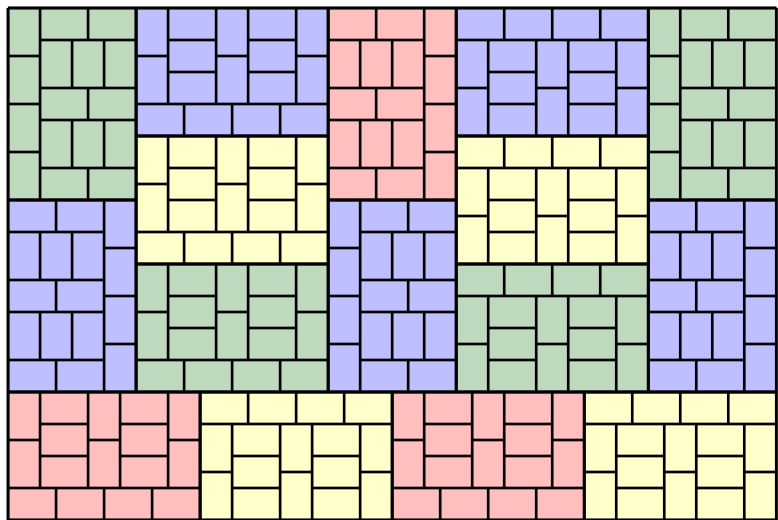
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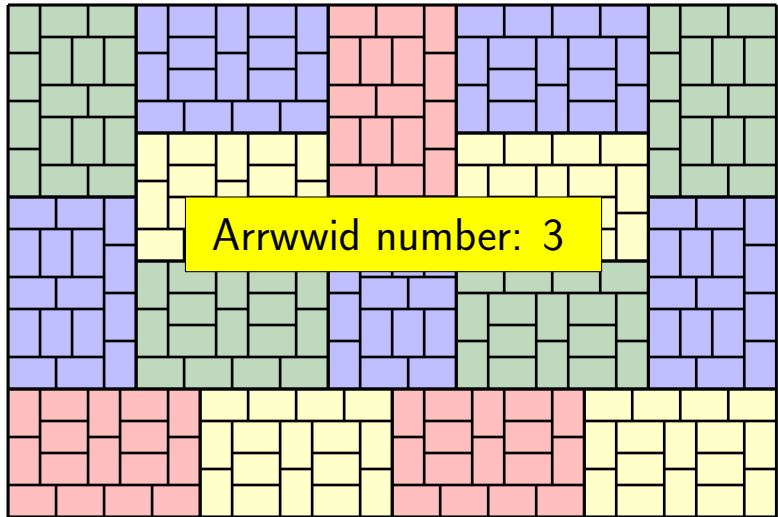
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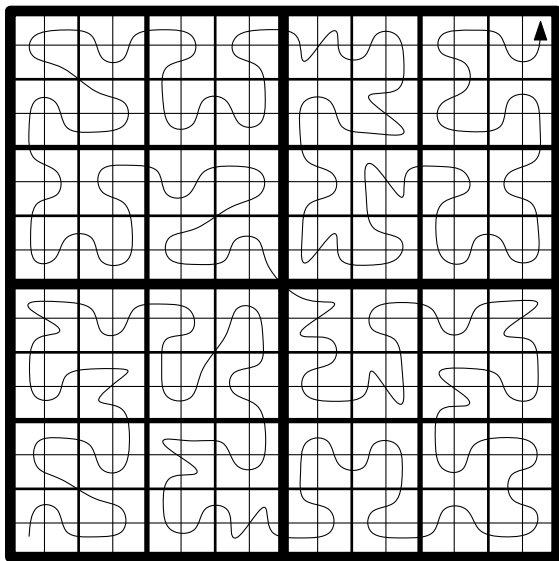
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R:



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The Arrwid number of a space-filling curve



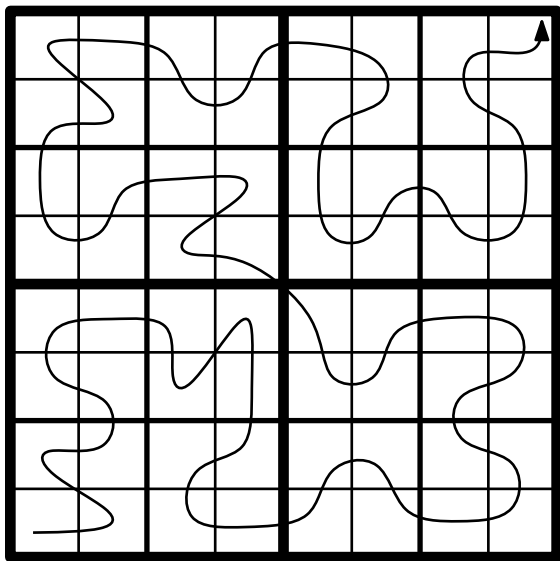
space-filling curve =
rec. tiling + ordering rule s.t.

all subtiles within any tile are
consecutive

← Example: AR^2W^2 order

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The Arrwid number of a space-filling curve



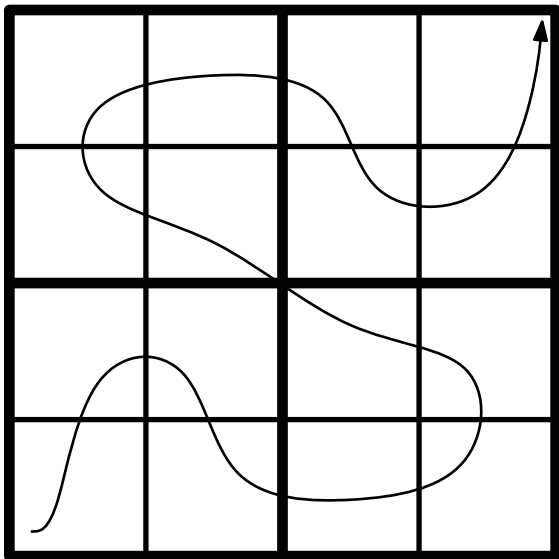
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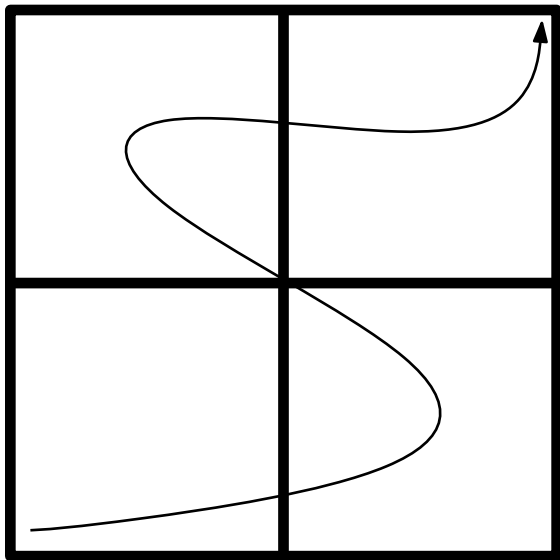


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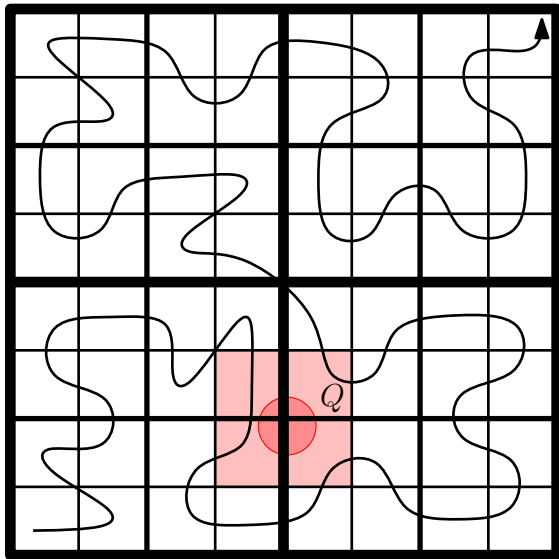
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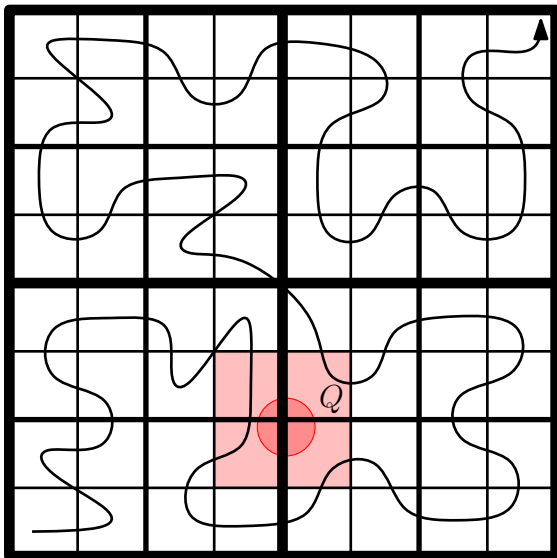
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The Arrwwid number of a space-filling curve

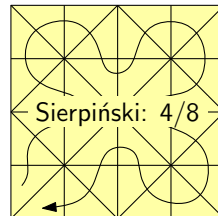
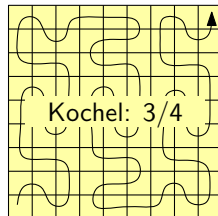
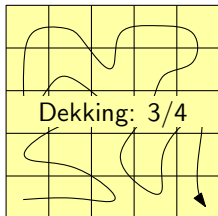
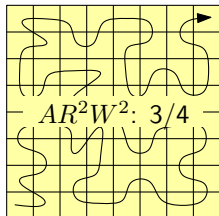
Definition for recursive tilings:

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Definition for space-filling curves (scanning orders of recursive tilings):

Arrwwid number = smallest a such that \exists constant c such that any disk Q is covered by $\leq a$ **sets of consecutive** tiles of total area $\leq c \cdot \text{area}(Q)$

Some curves have smaller Arrwwid nr. than the underlying tiling:



The Arrwwid number of a space-filling curve

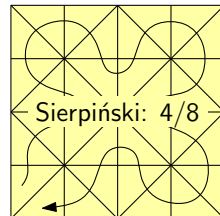
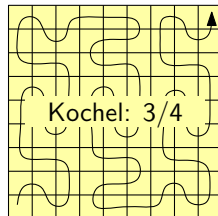
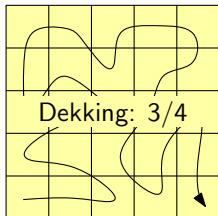
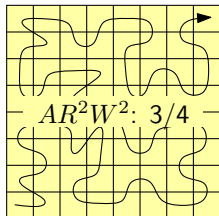
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Some tilings have Arrwwid nr. 3.

Some curves have Arrwwid nr. <3?

Asano et al.: Not if tiling divides squares into four squares
Yours truly: **Not ever** (assuming tiles are simply connected)

Results

Herman Haverkort: *Recursive tilings and space-filling curves with little fragmentation*.
arXiv:1002.1843 [cs.CG], 2010

best Arrwwid nrs. for tilings 2D

uniform squares 4 [ARRWW]

best nrs. for space-filling curves 2D

uniform squares 3 [ARRWW] ← most known curves have 4

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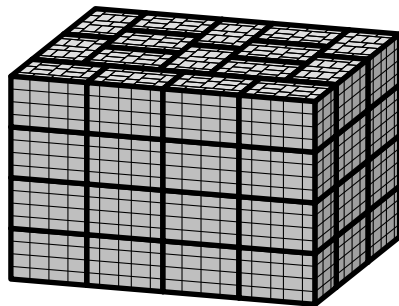
lower bound holds for simple tiles

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best Arrwwid nrs. for tilings	2D	3D	d -D
uniform hypercubes	4 [ARRWW]	8	2^d
uniform hyperboxes	3	6 (lwbd 4)	$\frac{3}{4} \cdot 2^d$ (lwbd:?)
uniform tiles of any shape	3		
<hr/>			
best nrs. for space-filling curves	2D		
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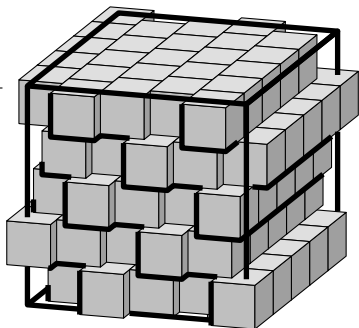
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	lower bound holds for simple tiles	...for convex tiles	(lwbd:?)

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lower bound holds for simple tiles ...for convex tiles

exponential gap between cubes / general shapes (but maybe also in c)

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THAT'S ALL FOLKS			
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